CONVERSION BETWEEN AUSTRIAN AND HUNGARIAN MAP PROJECTION SYSTEMS

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Abstract

Conversion between Austrian and Hungarian map projection systems is presented here. The conversion may be performed in two steps: first any kind of map projection systems should be transformed into WGS-84 ellipsoidal coordinates in one country, and then from WGS-84 ellipsoidal coordinates should be transformed into the desired system for the other country. A computer program has been developed to carry out all the possible transformation between the two countries. Using our method and software the transformation between Austrian and Hungarian map projection systems can be performed with a few centimeters accuracy for a few ten kilometers range of common border.

Keywords: map projection systems, transformation, WGS-84 ellipsoidal coordinates, GPS, Gauss-Krüger projection, Conversion between Austrian and Hungarian systems.

1. Introduction

Map projection systems and its reference surfaces, as well its triangulation networks differ in each countries. Conversions between countries are necessary if somebody want to use the own special map projection system in the neighbor country.

It is possible to make exact conversions between two map projection systems with closed mathematical expressions in cases only when both projection systems has the same reference surface and points of the same triangulation network coming from the same adjustment are represented in both projection systems. When any of the above mentioned requirements has not been met the conversion can be performed only using certain common points that have coordinates in both projection systems (HAZAY, 1964; VARGA 1981, 1982, 1986). In such case the accuracy of transformed coordinates depend on the reliability of triangulation networks and the position and number of selected common points. A slightly different coordinates will be resulted after the conversion process when another common points had been chosen. If there is no exact conversion using closed mathematical expressions between two map projection systems, the transformation can be performed only by Helmert transformation or polynomials up to the maximum degree five (*Rules for the Application of Unified National Projection,* 1975). Applying these methods we can eliminate the distortions of projection and the discrepancies of triangulation networks at the same process making a single plain transformation.

More precise and secure conversion can make using the so-called *mixed method,* when the transformation can be performed in two steps: first the distortions of projection and than the discrepancies of triangulation networks can be eliminated. In the first step we suppose that the two map projection systems have the same reference surface and the same triangulation network, and we perform the computation by the *coordinate method* using closed mathematical expressions (VARGA 1986). So in the first step we get approximated plane coordinates in the second projection system. Then in the second step we perform a transformation by polynomials using common points. The common points for determining the coefficients of these transformation polynomials should be that points, which have both the previously computed approximated values and the original plane coordinates in the second projection system. We can use transformation polynomials having lower degrees in the second step of transformation to eliminate of discrepancies of the different triangulation networks, against if we would make the conversion in only one step using power series.

2. Conversion between Austrian and Hungarian systems

Conversion between Hungarian and Austrian map projection systems can't be executed by *coordinate method* using closed mathematical expressions because the position and orientation of reference surfaces are slightly different, and the triangulation networks had been adjusted one by one – although there is the Bessel's ellipsoid as a reference surface of projection systems which is applied in Hungary and Austria too, and there are some common points of different triangulation networks. So the conversion between the two countries can be performed only by transformation polynomials using common points.

Map projection systems of neighboring countries generally can be expanded only for a few ten kilometers range from the common border because common points can always be found only in this region. GPS is the most powerful tool for making common points anywhere, because determining of *X, Y, Z* spatial geocentric Cartesian, or WGS-84 coordinates of points of triangulation network, we can create such system of common points which are very suitable for conversion of map projection system between the countries.

Having enough common points made by GPS afford possibility to make a conversion between map projection systems of Hungary and Austria. So it is all the same, to transform coordinates between map projection systems of Hungary and Austria with different reference surfaces (Bessel's ellipsoid in Austria, and Bessel's, Krassovky's or IUGG-67 ellipsoids in Hungary) and different meridian of origin (prime meridian of Ferro for Austria and prime meridian of Greenwich for Hungary).

Transformations between all existing Hungarian map projection systems were completed earlier (VÖLGYESI at all, 1996) and there are very precise transformations from all Hungarian map projection systems into WGS-84 or *X, Y, Z* spatial geocentric Cartesian systems (VÖLGYESI, 1997). If we want to convert coordinates between Hungary and Austria then the next important task is to make transformations between WGS-84 and the other map projection systems used in Austria.

3. Practical solution

Conversion between coordinates in *Table 1* is performed by the conversion program in the area of Hungary and Austria in 213 combinations as it is enlisted in *Table 2*.
South cylindrical projection system (HDR) and Budapest city stereographic projection (VST) are not to be found on the above list because the regions where these two Hungarian map projection system are used, is not neighboring to Austria and using these two systems there is no practical need to make conversion between Hungary and Austria.

Table 1. Hungarian and Austrian map projection systems

VTN	System without projection in Hungary
BES	Bessel's Ellipsoidal
SZT	Budapest Stereographic Projection
KST	Hungarian Military Stereographic Projection
HER	Hungarian North Cylindrical System
HKR	Hungarian Middle Cylindrical System
ABE	Austrian Bessel's Ellipsoidal
AGK	Austrian Gauss-Krüger Projection
IUG	Hungarian IUGG-67 Ellipsoidal
EOV	Hungarian Unified National Projection
KRA	Hungarian Krassovsky's Ellipsoidal
GAK	Hungarian Gauss-Krüger Projection
WGS	WGS-84 Ellipsoidal /GPS/
XYZ	Spatial Cartesian Geocentric /GPS/
UTM	Universal Transverse Mercator

Table 2. Combination of transformations

	VTN	BES	SZT	KST	HER	HKR	ABE	AGK	IUG	EOV	KRA	GAK	WGS	XYZ	UTM
VTN	–	×	×	×	×	×	×	×	×	×	×	×	×	×	×
BES	×	–	+	+	+	+	×	×	×	×	×	×	×	×	×
SZT	×	+	–	+	+	+	×	×	×	×	×	×	×	×	×
KST	×	+	+	–	+	+	×	×	×	×	×	×	×	×	×
HER	×	+	+	+	–	+	×	×	×	×	×	×	×	×	×
HKR	×	+	+	+	+	–	×	×	×	×	×	×	×	×	×
ABE	×	×	×	×	×	×	–	+	×	×	×	×	×	×	×
AGK	×	×	×	×	×	×	+	!+!	×	×	×	×	×	×	×
IUG	×	×	×	×	×	×	×	×	–	+	×	×	×	×	×
EOV	×	×	×	×	×	×	×	×	+	–	×	×	×	×	×
KRA	×	×	×	×	×	×	×	×	×	×	–	+	×	×	×
GAK	×	×	×	×	×	×	×	×	×	×	+	!+!	×	×	×
WGS	×	×	×	×	×	×	×	×	×	×	×	×	–	+	+
XYZ	×	×	×	×	×	×	×	×	×	×	×	×	+	–	+
UTM	×	×	×	×	×	×	×	×	×	×	×	×	+	+	!+!

Table 2 conveys us information on the possibility and accuracy of conversions very simply. Double lines in *Table 2* separate map projection systems belonging to different reference surfaces. (By reference surface the ellipsoid is meant, though the fact should be acknowledged that the approximating /Gaussian/ sphere serves also as a reference surface for those map projection systems where a double projection is applied and an intermediate sphere is the reference surface at the second step of the projection to get coordinates on a plane or on a plane developable surface. Coordinates on this approximating sphere have no practical role for users.)

Plus "+" signs at the intersection fields of rows and columns indicate that an exact conversion between the two map projection system is possible using closed mathematical formulas found in reference works of (HAZAY, 1964) and (VARGA, 1981, 1986) for transformation. In this case the accuracy of transformed coordinates is the same as the accuracy of coordinates to be transformed.

Cross "×" signs in *Table 2* indicate the impossibility of transformation between the two map projection systems with closed mathematical formulas and the conversion –

according to rules found in [2] is performed using polynomials as of a finite (maximum five) degree with limited accuracy (VÖLGYESI at all, 1996; VÖLGYESI, 1997).

Minus "–" signs in *Table 2* are reminders of the fact that an identical (transformation into itself) conversion has no meaning except of the Gauss-Krüger and UTM projection systems where the need of conversion between different zones frequently arises. Hence a "!+!" sign indicates that it is possible to make exact conversions between different zones of the Gauss-Krüger and UTM map projection systems.

The conversion logic between the different map projection systems can be overviewed on *Fig. 1*.

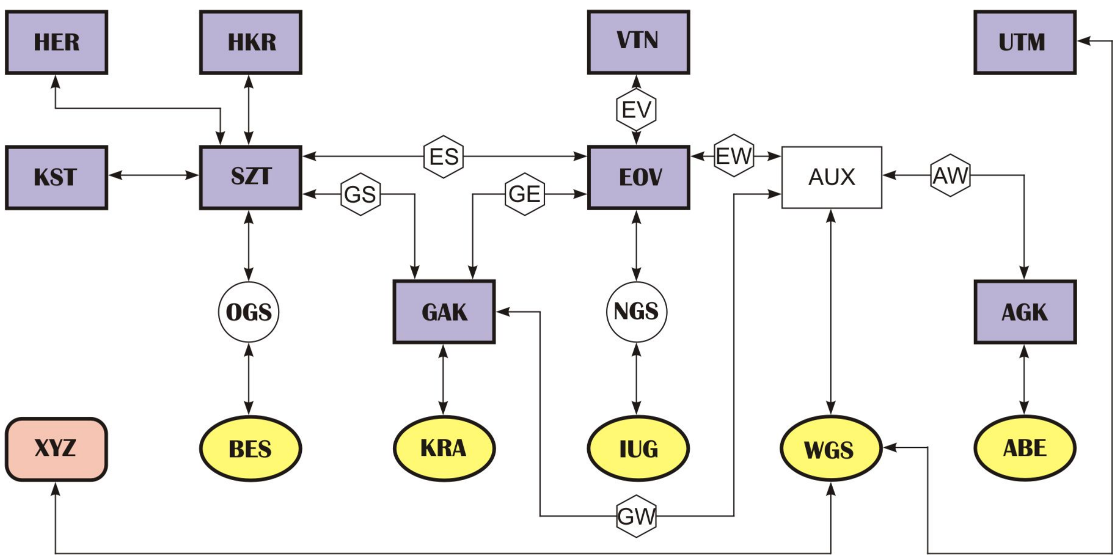

Fig. 1. Conversion flow between different map projection systems

Transformation paths – and their directions – between different systems are pictured by arrows. It can be seen that it is possible to convert between both WGS-84 ↔ Unified National Projection (EOV) and WGS-84 ↔ Gauss-Krüger systems only through other intermediate systems. E.g. if a conversion between WGS and EOV systems is needed then WGS-84 coordinates first have to be converted into a so-called auxiliary system (AUX) and finally they should be converted from this AUX system into EOV coordinates; or e.g. if a conversion between GAK and WGS systems is needed then Gauss-Krüger coordinates first has to be converted into an auxiliary system (AUX) and finally they should be converted into the WGS-84 ellipsoid.

If any two systems in *Fig. 1* are connected through a hexagonal block then between these two systems only an approximately accurate conversion could be made by transformation polynomials. In *Fig. 1* the two-letters abbreviations in hexagonal blocks show which data files, containing transformation polynomials, have to be used to convert between the two neighboring systems. If any two systems in *Fig. 1* are connected by a continuous line then an exact conversion by the coordinate method, i.e. through closed mathematical expressions can be made.

Since it may cause problems even for experts to apply correct methods of conversion between a multitude of map projection systems so we worked out such a software by which conversions can be made between Hungarian and Austrian map projection systems and their reference coordinates in all combination, the usage of which can cause no problem even for users having no deeper knowledge in map projections.

4. Initial data

In cases of any two systems in *Fig. 1* are connected through a hexagonal block the conversion could only be made by transformation polynomials using common points. E.g. this is the case of Austrian Gauss-Krüger and Spatial Cartesian Geocentric /XYZ/ or WGS-84 systems.

Between Austrian Gauss-Krüger and Spatial Cartesian Geocentric /XYZ/ systems 64 common points were used to determine the coefficients of transformational polynomials for the complete area of Austria. The *X, Y, Z* Spatial Cartesian Geocentric coordinates, served by GPS measurements are referring to *ITRF94* (for 1993 epoch).

5. Transformation between WGS-84 and Austrian Gauss-Krüger systems

A simple conversion is possible by closed mathematical formulas, between Spatial Cartesian Geocentric (XYZ) and WGS-84 systems. GPS can serve both XYZ and WGS-84 coordinates. Transformation between WGS-84 and Austrian Gauss-Krüger systems can be completed in two steps: first WGS-84 coordinates have to be converted into an auxiliary plain system (AUX), and the next step is the conversion from this auxiliary plain system into the Austrian Gauss-Krüger system using polynomials – as it can be seen in *Fig. 1*. The first step can be computed by simple closed mathematical formulas (VARGA, 1986), but the second step can be completed by maximum five-order polynomials depending on the number of common points [2]. For example, the connection between *x, y* coordinates of the projection system *I.* and x', y' coordinates of the projection system *I.* is established by the

$$\begin{aligned} x' = {} & A_0 + A_1 x + A_2 y + A_3 x^2 + A_4 xy + A_5 y^2 + A_6 x^3 + A_7 x^2 y + A_8 xy^2 + A_9 y^3 + \\ & A_{10} x^4 + A_{11} x^3 y + A_{12} x^2 y^2 + A_{13} xy^3 + A_{14} y^4 + A_{15} x^5 + A_{16} x^4 y + A_{17} x^3 y^2 + \\ & A_{18} x^2 y^3 + A_{19} xy^4 + A_{20} y^5 + \ldots \end{aligned} \qquad (1a)$$

$$\begin{aligned} y' = {} & B_0 + B_1 x + B_2 y + B_3 x^2 + B_4 xy + B_5 y^2 + B_6 x^3 + B_7 x^2 y + B_8 xy^2 + B_9 y^3 + \\ & B_{10} x^4 + B_{11} x^3 y + B_{12} x^2 y^2 + B_{13} xy^3 + B_{14} y^4 + B_{15} x^5 + B_{16} x^4 y + B_{17} x^3 y^2 + \\ & B_{18} x^2 y^3 + B_{19} xy^4 + B_{20} y^5 + \ldots \end{aligned} \qquad (1b)$$

polynomials. Coefficients $A_0 - A_{20}$ and $B_0 - B_{20}$ (altogether 42 coefficients) can be determined by using common points suitably through an adjustment process.

An important question is to determine the optimal degree of the polynomial. By considering a simple way of reasoning one could arrive at the conclusion that the higher the degree of the polynomial the higher the accuracy of map projection conversions will be. On the contrary, it could be proved by our tests that the maximum accuracy was resulted by applying five degree polynomials. No matter whether the degree was decreased or increased, the accuracy of transformed coordinates was lessened alike (more considerably by decreasing, less considerably by increasing – while the biggest discrepancies can be found at the edges of networks).

6. Accuracy of conversion

It is possible to convert through closed mathematical expressions between certain map projection systems. In these cases the accuracy of transformed plane coordinates is equal to the accuracy of initial coordinates (1 mm or 0.0001"). These conversions are referred to in *Table 2* with "+" and "!+!" signs or these systems are connected by continuous lines (arrows) in *Fig. 1*.

In all other cases when the transformation path between any two systems passes through a hexagonal block (or blocks), the accuracy of transformed coordinates depends, on one side, how accurately the control networks of these systems fit into each other; and on the other side, how successful was the determination of transformation polynomial coefficients. It follows also from these facts that no matter how accurately these transformation polynomial coefficients was determined, if the triangulation networks of these two systems do not fit into each other accurately – since there were measurement, adjustment and other errors during their establishment – then certainly no conversion of unlimited accuracy can be performed (in other terms, only such an accurate conversion between two map projection systems is possible that the accuracy allowed by the determination errors or discrepancies of these control networks). This fact, of course does not mean that ones should not be very careful when the method of transformation is selected or – when the polynomial method is applied – the coefficients are determined.

So accuracy of transformation can be described by the following logic: Coefficients of transformation polynomials (1) should be first computed based on coordinates of common points y_i, x_i and y_i', x_i' in systems *I* and *II*, respectively. Then y_i, x_i coordinates in system *I* can be transformed into coordinates ty_i', tx_i' in system *II* by using these coefficients. Finally the standard error characteristic to conversion,

$$\mu = \sqrt{\frac{\sum_{i=1}^{n} \Delta y_i^2 + \sum_{i=1}^{n} \Delta x_i^2}{n}} \tag{2}$$

can be determined, where

$$\begin{aligned} \Delta y_i &= ty_i' - y_i' \\ \Delta x_i &= tx_i' - x_i' . \end{aligned} \tag{3}$$

Using polynomial method and applying expression (2) standard errors are summarized between Hungarian systems for the complete area of Hungary in *Table 3*.

Table 3. Standard errors of polynomial method

Hungarian systems	Number of common points	Standard error
EOV – SZT	162	±0.247 *m*
EOV – WGS	43	±0.050 *m*
EOV – GAK	79	±0.102 *m*
EOV – VTN	27	±0.046 *m* *
GAK – WGS	34	±0.084 *m*
GAK – SZT	184	±0.046 *m*

* valid only for territory *Baranya*

With a view to transformation between Austrian and Hungarian map projection systems the two most important Hungarian transformations are the EOV–WGS-84 and the Hungarian Gauss-Krüger–WGS-84. The contour line map of standard errors defined by Eq. (2) for these two systems can be seen in *Fig. 2* and *Fig. 3* respectively.

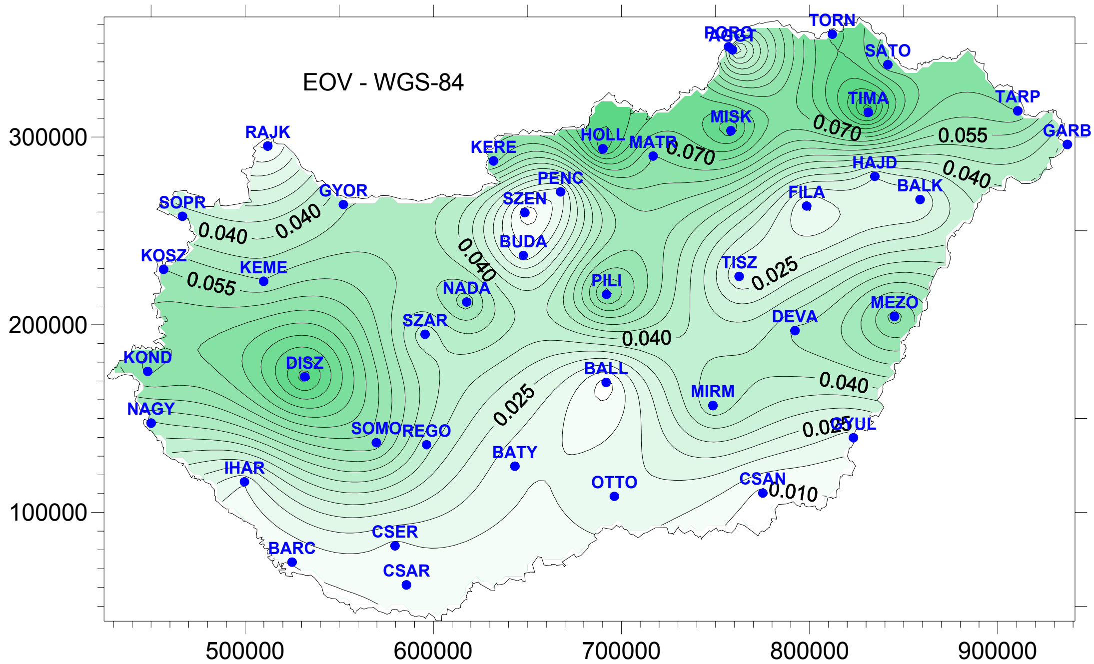

Fig. 2. Standard errors of EOV–WGS-84 transformation (contour labels in [m])

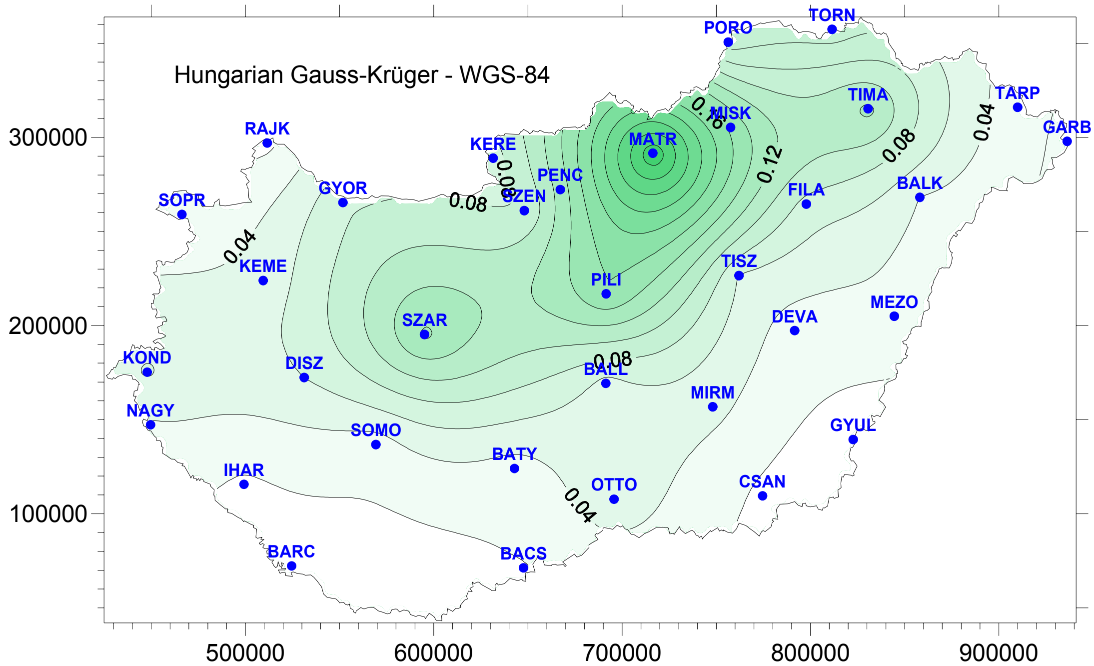

Fig. 3. Standard errors of Gauss-Krüger–WGS-84 transformation (contour labels in [m])

Our experiences showed the fact that although the accuracy can somewhat be increased by increasing the number of common points within the polynomial method but the accuracy of conversion can not be increased beyond a certain limit even with this method since there is a difference between the two triangulation networks. In certain cases, however, an improvement could be gained when transformation polynomial coefficients are not determined for the complete area of the country but for only smaller region common points are given and transformation polynomial coefficient are determined. In such cases conversions, of course, must not be made outside the sub-area where the coefficients of transformation polynomials were determined, and the junction of these regions is not a simple problem.

Table 4. Δy and *Δx* differences between the original and the transformed coordinates in five different versions of the Austrian Gauss-Krüger–WGS-84 transformation.

	Version 1		**Version 2**		**Version 3**		**Version 4**		**Version 5**		
Point	**Δy**	**Δx**	**Δy**	**Δx**	**Δy**	**Δx**	**Δy**	**Δx**	**Δy**	**Δx**	
AGGS	-5.207	4.165	-3.633	6.321	-0.579	-0.295	0.376	0.256	0.108	0.089	
BRBG	-6.819	12.798	-4.732	10.556	0.043	0.212	0.010	0.193	-0.059	-0.029	
ERZK	0.736	-7.803	-0.093	-2.773	-1.514	0.304	-1.748	0.169	-0.046	0.054	
FORC	7.819	-5.195	2.906	-6.926	-0.515	0.484	-0.948	0.234	-0.006	0.171	
FRAU	6.314	-1.885	1.786	-3.630	0.332	-0.482	0.613	-0.320	0.028	-0.022	
GRAZ	-4.945	4.354	-3.388	5.033	-1.166	0.219	-1.380	0.095	0.109	-0.128	
GSST	-0.263	-1.652	0.329	-1.654	-0.368	-0.145	-0.313	-0.113	-0.045	-0.057	
GUES	-11.192	-3.557	-0.656	1.775	0.242	-0.171	0.680	0.081	0.139	0.083	
HAID	2.387	-6.658	4.778	-7.263	2.256	-1.800	0.904	-2.580	-	-	④
HOLL	-3.696	0.008	-0.671	3.013	0.456	0.573	0.256	0.457	0.063	0.021	
HUTB	-2.492	1.381	-0.822	4.500	0.716	1.169	-0.313	0.576	-0.033	0.033	
HUTS	-3.814	3.785	-4.268	5.488	-1.750	0.034	-0.394	0.816	0.045	0.034	
HZBG	5.299	-7.787	3.811	-7.711	0.034	0.470	-0.474	0.177	0.032	-0.081	
KULM	-5.728	2.507	-3.557	4.856	-1.456	0.305	-1.800	0.107	-0.030	-0.107	
LUNZ	-3.894	4.493	-4.432	4.425	-1.783	-1.314	0.273	-0.128	0.027	-0.051	
OGDF	-2.425	-2.234	-1.503	0.564	-0.996	-0.535	0.088	0.090	-0.011	-0.010	
RADB	11.245	-1.456	2.479	-4.221	0.480	0.109	0.118	-0.100	-0.068	-0.072	
RETZ	11.835	-14.820	7.900	-15.073	0.487	0.985	-0.246	0.563	-0.089	-0.069	
RIEG	-4.770	9.798	-3.473	5.625	-0.884	0.018	-0.554	0.209	-0.078	0.072	
SLAG	-3.826	11.965	-4.148	7.290	-0.472	-0.672	0.190	-0.290	0.047	0.007	
TEIA	3.054	-6.412	7.154	2.046	8.398	-0.650	7.881	-0.949	-	-	④
TIRK	-1.868	-5.679	-1.814	-1.327	-2.074	-0.763	-0.841	-0.052	-0.122	-0.014	
WIEN	1.318	-2.255	3.533	-3.912	1.137	1.278	-0.050	0.594	0.011	-0.013	
ALTF	20.783	12.659	0.752	-1.984	0.000	-0.356	0.284	-0.192	-0.014	-0.188	
ASTN	-14.986	64.319	-20.994	45.474	-	-	-	-	-	-	②
DAST	10.985	41.737	-1.093	-0.392	-1.086	-0.406	-0.283	0.057	0.023	0.025	
EBRI	-276.222	-217.304	-	-	-	-	-	-	-	-	①
EDLW	8.759	40.935	-1.969	5.797	0.614	0.201	0.062	-0.118	-0.045	-0.145	
FRBS	47.547	35.478	0.743	-1.740	0.077	-0.295	0.400	-0.109	0.089	-0.043	
GABL	-7.337	-6.390	-1.309	1.542	-0.905	0.665	-1.644	0.239	0.020	0.018	
GERL	6.382	-6.952	3.329	-6.577	0.337	-0.096	0.344	-0.092	0.036	-0.044	
GOLL	25.205	50.856	4.215	-13.209	-1.445	-0.948	0.336	0.080	-0.002	0.065	
GRMS	-3.47	1.010	1.514	-1.297	0.695	0.475	-0.112	0.010	-0.038	0.008	
GUBG	25.054	38.135	12.896	7.122	12.781	7.370	-	-	-	-	③
HEMB	48.714	36.038	0.844	-2.321	-0.192	-0.076	0.202	0.151	-0.093	0.239	
HOPY	-2.026	11.027	-5.740	3.999	-2.924	-2.101	0.170	-0.317	0.017	-0.034	
HSHN	22.765	-28.274	16.657	-40.446	-1.582	-0.940	0.121	0.042	0.002	0.089	
HUST	18.270	56.332	-0.152	0.529	0.027	0.141	0.071	0.167	-0.074	0.066	
LEND	-75.005	-274.003	-	-	-	-	-	-	-	-	①
LOIB	58.347	54.196	-2.034	5.042	0.288	0.012	0.073	-0.112	0.244	-0.171	
MAGD	13.985	0.638	2.949	-6.055	0.154	-0.002	0.307	0.086	0.152	0.110	
MAYB	6.714	3.420	0.761	-9.518	-2.726	-1.965	-0.020	-0.405	-0.016	-0.157	
MOAH	21.044	57.190	0.238	0.387	0.382	0.075	0.359	0.062	0.028	-0.027	
OBWG	-63.438	-172.519	-	-	-	-	-	-	-	-	①
OSWA	-10.794	19.040	-10.905	18.385	-1.971	-0.968	-0.399	-0.061	-0.051	0.056	
PLAN	-1.958	14.444	-2.103	2.029	-1.199	0.072	-1.173	0.086	0.016	0.003	
RADS	4.508	29.264	-0.665	1.998	-0.002	0.562	-0.573	0.233	-0.052	0.119	
ROSF	27.099	50.343	5.512	-15.563	-1.251	-0.915	0.412	0.044	0.038	0.002	
SEBS	76.846	69.086	-2.347	5.853	0.126	0.495	-0.493	0.138	-0.023	0.107	
SNBG	-0.734	-16.842	4.014	-4.059	1.598	1.174	-0.058	0.219	-0.008	-0.004	
SOBO	31.811	24.058	0.484	-2.563	-0.511	-0.408	0.059	-0.079	-0.088	-0.032	
STAL	-1.955	15.111	-3.081	7.389	0.389	-0.127	0.475	-0.077	0.399	0.044	
TILL	-14.967	-11.343	-0.556	1.191	-0.041	0.076	0.017	0.109	0.311	0.115	
TPLZ	7.231	33.067	-2.600	-0.191	-2.142	-1.184	-0.354	-0.153	0.023	-0.060	
TREH	-7.356	-6.040	-0.999	0.826	-0.696	0.168	-0.573	0.239	-0.336	0.138	
VILA	5.886	-2.320	1.369	-3.535	-0.176	-0.189	0.015	-0.079	-0.255	-0.062	
WANS	-1.300	16.044	-1.414	4.411	0.560	0.135	0.090	-0.136	-0.025	-0.074	
DMBL	-1.571	-38.326	9.777	-19.402	0.570	0.539	-0.353	0.007	0.105	-0.085	
FLEX	-2.687	16.559	-2.769	6.142	0.082	-0.033	0.093	-0.027	0.055	-0.102	
KRAH	-2.837	-0.005	-2.512	5.179	-0.112	-0.020	0.027	0.060	0.014	0.142	
KRAI	-2.813	0.013	-2.503	5.158	-0.117	-0.010	0.023	0.070	0.008	0.151	
NOSL	5.528	24.739	-5.320	9.943	-0.512	-0.472	-0.158	-0.268	-0.413	-0.112	
OBGL	6.798	-23.023	7.162	-15.510	-0.042	0.095	-0.111	0.055	-0.038	-0.072	
PFAN	2.124	-0.255	0.369	-1.038	-0.074	-0.080	0.038	-0.015	-0.032	0.006	
	± 68.328 m		*± 11.616 m*		*± 2.530 m*		*± 1.251 m*		***± 0.152 m***		

The next question is the accuracy of transformation between Austrian Gauss-Krüger and WGS-84 systems. We summarized the results of our test computations in *Table 4*. There are Δy and Δx differences between the original and the transformed coordinates in five different versions for each common points computed by (3), and there is the standard error characteristic to different versions of conversion computed by (2) in the last row of *Table 4*.

In the case of *version 1* all the given *64* common points between Austrian Gauss-Krüger and WGS-84 systems for the complete area of Austria were used for determining the coefficients of transformation polynomials (1). Using these coefficients, WGS-84 coordinates was transformed into Gauss-Krüger system, and the differences of original and transformed Gauss-Krüger coordinates are listed at the 2.nd and 3.rd columns of *Table 4*. There are surface views of these differences in *Fig. 4*, the "surface heights" are $\sqrt{\Delta y^2 + \Delta x^2}$ on the figure. There are 3 points (*EBRI*, *LEND* and *OBWG*) in which very big errors (a few hundred meters differences) can be found. The standard error characteristic to transformation of *version 1* is ±68.328 m. Probably the GPS stations were not set up correctly to that places, where Gauss-Krüger coordinates are referred. So these three points were canceled from the next versions of computations.

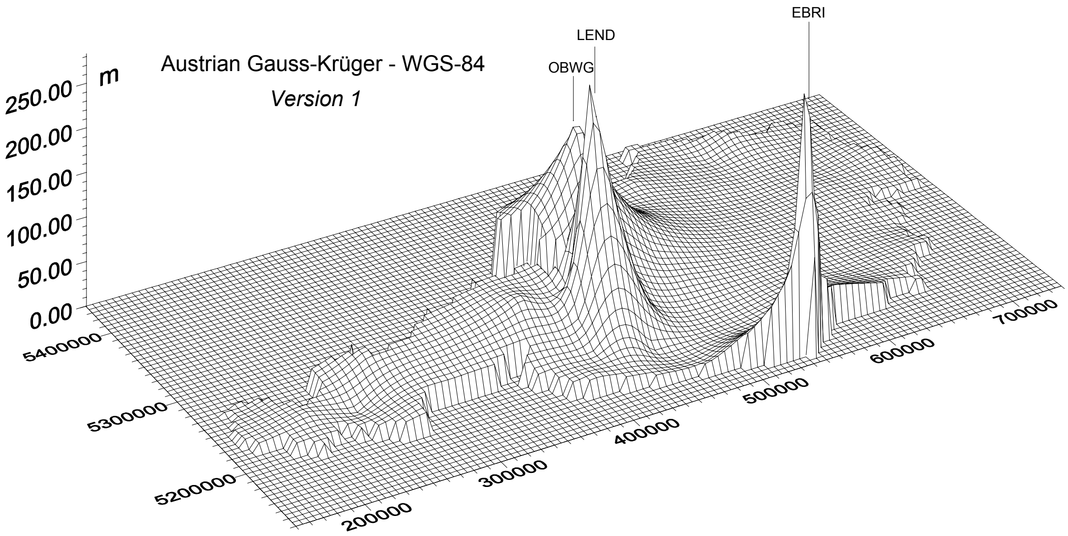

*Fig. 4. S*urface view of differences between the original and the transformed coordinates in the *version 1* of the Austrian Gauss-Krüger–WGS-84 transformation

In the *version 2* the remaining *61* common points were used for determining the coefficients. Using these values for transformation the differences of coordinates are listed at the 4.th and 5.th columns of *Table 4* and the surface view of these differences can be seen in *Fig. 5.* In *version 2* there is 1 point (*ASTN*) in which too big error *50 m* difference can be found. The standard error of *version 2* is ±11.616 m. So the point *ASTN* was canceled from the next versions of computations.

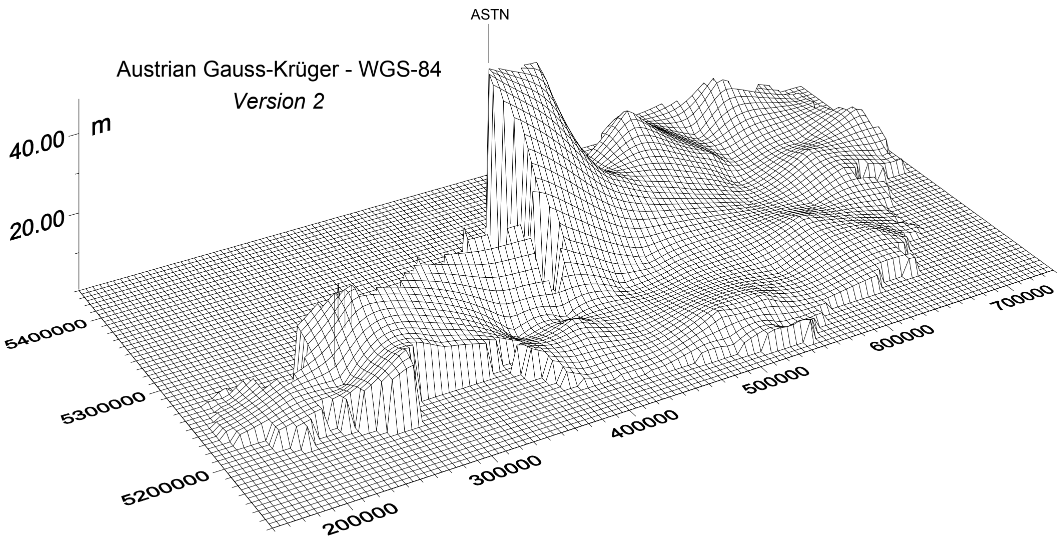

Fig. 5. *S*urface view of differences between the original and the transformed coordinates in the *version 2* of the Austrian Gauss-Krüger–WGS-84 transformation

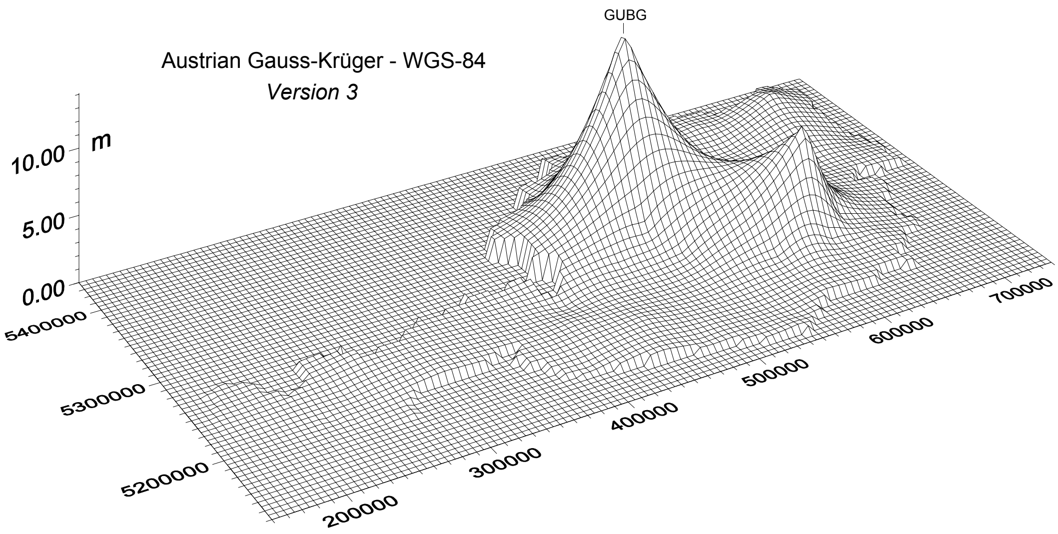

Fig. 6. *S*urface view of differences between the original and the transformed coordinates in the *version 3* of the Austrian Gauss-Krüger–WGS-84 transformation

In the *version 3* the remaining 60 common points were used for determining the coefficients. Using these coefficients for transformation the differences of coordinates are listed at the 6.th and 7.th columns of *Table 4* and the surface view of these differences can be seen in *Fig. 6.* In *version* 3 there was 1 point (*GUBG*) in which nearly 15 m difference can be found. The standard error of *version 3* is ±2.530 m. So this point was canceled from the next versions of computations.

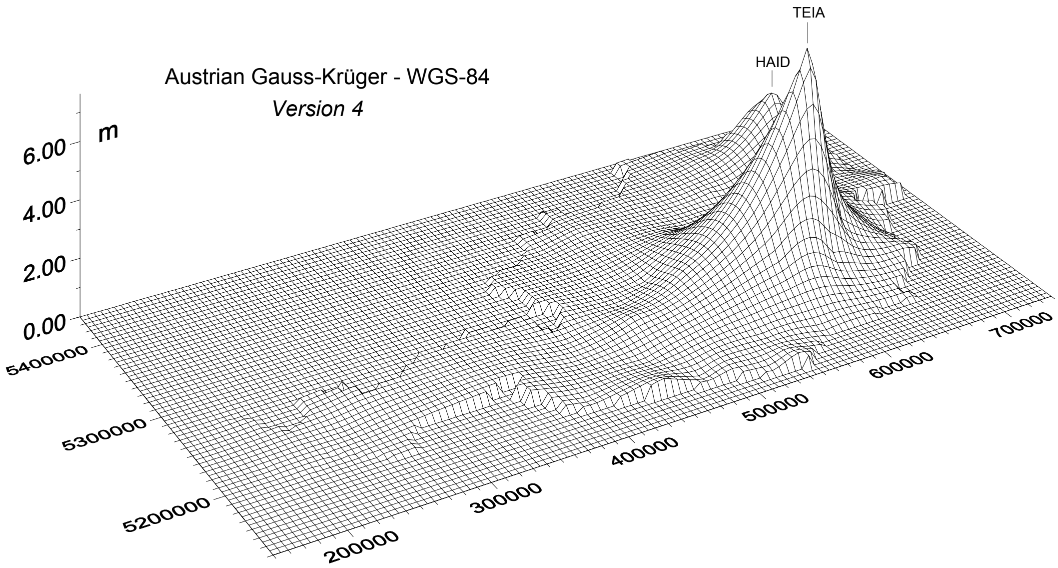

*Fig. 7. S*urface view of differences between the original and the transformed coordinates in the *version 4* of the Austrian Gauss-Krüger–WGS-84 transformation

In the *version 4* the remaining *59* common points were used for determining the coefficients. Using these coefficients for transformation the differences of coordinates are listed at the 8.th and 9.th columns of *Table 4* and the surface view of these differences can be seen in *Fig. 7.* In *version 4* there were 2 points (*HAID* and *TEIA*) in which a few meters differences can be found, and the standard error of *version 4* is ±1.251 m. These two points were canceled from the last version of computations.

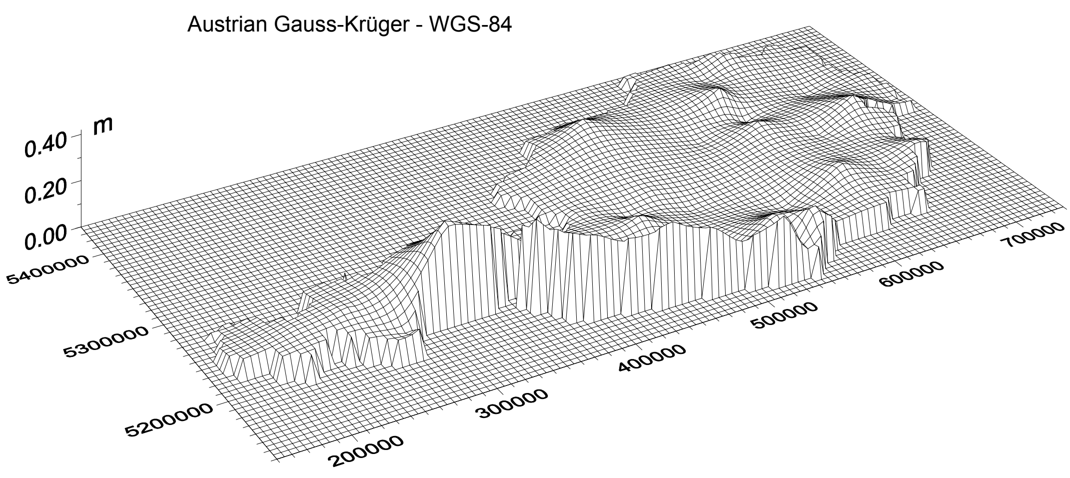

*Fig. 8. S*urface view of differences between the original and the transformed coordinates in the *version 5* of the Austrian Gauss-Krüger–WGS-84 transformation

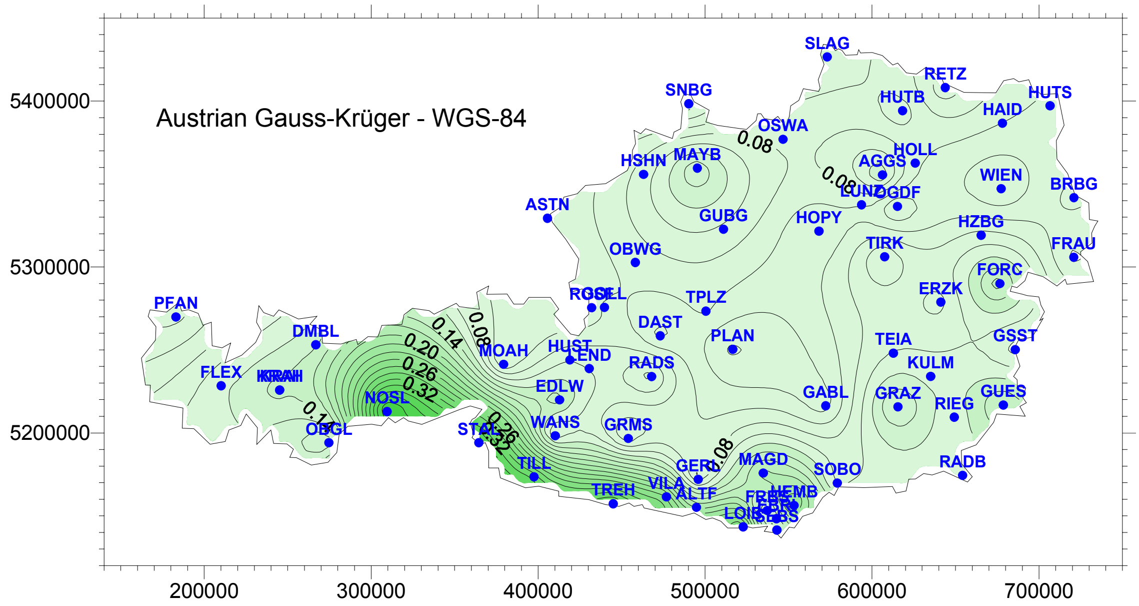

Fig. 9. Standard errors of Austrian Gauss-Krüger–WGS-84 transformation (contour labels in [m])

In the case of *version 5* the remaining 57 common points were used for determining the coefficients of transformation polynomials. Using these coefficients, for the complete area of Austria in general a few centimeters, maximum 4 decimeters differences can be found, and the standard error of transformation is ±0.152 m between Austrian Gauss-Krüger and WGS-84 systems. The differences of coordinates are listed at the 10.th and 11.th columns of *Table 4*. The surface view and the contour line map of these differences can be seen in *Fig. 8* and *Fig. 9* respectively.

Using the coefficients of transformation polynomials of *version 5*, the discrepancies of original and transformed Gauss-Krüger coordinates of the seven canceled common points are listed in *Table 5*.

Table 5. Discrepancies of original and transformed Gauss-Krüger coordinates of the seven canceled common points

	$\Delta y\,[m]$	$\Delta x\,[m]$	$\sqrt{\Delta y^2 + \Delta x^2}$
EBRI	-316.45	-290.78	429.76
LEND	-65.29	-329.96	336.36
OBWG	-73.62	-212.24	224.65
ASTN	-76.20	136.73	156.53
GUBG	15.80	11.49	19.54
TEIA	10.16	-0.80	10.19
HAID	1.18	-3.53	3.72

The gross errors in the first four points (*EBRI, LEND, OBWG, ASTN*) indicate, the GPS stations were not set up correctly to that places, where Gauss-Krüger coordinates are referred, and it may be justified to omit them from the common points.

The explanation of discrepancies in the remaining three points in *Table 5* is uncertain. It may be important to investigate whether the problems are local or referring for bigger surroundings of points *GUBG, TEIA* and *HAID* – so the GPS measurements should be controlled and repeated here.

If the problem is local, the reason might be the same as in the case of the first four points, and it may be justified to omit them from the common points too – or to replace them by exact new values.

If the problems are referring for bigger surroundings of these three points, the reason might come from not too precise earlier triangulation measurements and/or wrong adjustment of Gauss-Küger control network points.

In this case it would be made a denser net of common points in the vicinity of few ten kilometers of points *GUBG, TEIA* and *HAID*, and it would be necessary to determine new coefficients of transformation polynomial for the surroundings of these 3 points one by one. So, the transformation for the whole country will not be damaged by the points *GUBG, TEIA* and *HAID*, but the coordinates could be transformed with a suitable accuracy at the vicinity of these points at the same time using the local coefficients of transformation polynomial.

Table 6. Accuracy of conversion in common points

AGK – WGS			**GAK – WGS**			**EOV – WGS**		
Point	Δy	Δx	Point	Δy	Δx	Point	Δy	Δx
FRAU	0.028	-0.022	RAJK	-0.016	0.003	RAJK	-0.001	0.008
FORC	-0.006	0.171	SOPR	0.022	-0.013	SOPR	-0.023	-0.029
GSST	-0.045	-0.057	KOND	-0.060	0.014	KOSZ	0.045	0.031
GUES	0.139	0.083				KOND	-0.046	-0.048
±0.124			±0.039			±0.045		

Concerning the transformation between Austrian and Hungarian map projection systems, there is a remarkable accuracy of conversion for a few ten kilometers range of common border. Accuracy of conversion between the two countries can be characterized based on the accuracy of conversion of points in the vicinity of the common border. Accuracy of conversion of common points next to the border is summarized in *Table 6*. It can be seen, that mean error of conversion between Austrian Gauss-Krüger and WGS-84 systems based on 4 points next to the Hungarian border is ±0.124m, mean error of conversion between Hungarian Gauss-Krüger and WGS-84 systems based on 3 points next to the Austrian border is ±0.039m, and mean error of conversion between Hungarian EOV and WGS-84 systems based on 4 points next to the Austrian border is ±0.045m. So the final conclusion may be that using our method and software for the given common points, the transformation between Austrian and Hungarian map projection systems can be performed with a few centimeters accuracy for a few ten kilometers range of common border.

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References

[1] HAZAY I. (1964): *Map projections*. Tankönyvkiadó, Budapest (in Hungarian)
[2] *Rules for the Application of Unified National Projection* (1975). MÉM OFTH, Budapest (in Hungarian)
[3] VARGA J. (1981): *New Methods of Conversion Between our Projection Systems*. Budapest, Technical doctoral dissertation (in Hungarian).
[4] VARGA J. (1982): *Conversion between the Unified National Projection (EOV) and between our Former Projections*. Geodézia és Kartográfia No2. (in Hungarian)
[5] VARGA J. (1986): *Control Networks I.* (Map projections). Tankönyvkiadó, Budapest (in Hungarian)
[6] VÖLGYESI L., Gy. TÓTH, J. VARGA (1996): *Conversion between Hungarian map projection systems.* Periodica Polytechnica Civil Eng. Vol. 40, No. 1, pp. 73-83.
[7] VÖLGYESI L., (1997): Transformation of Hungarian Unified National and Gauss-Krüger Projection System into WGS-84. *Reports on Geodesy, Warsaw University of Technology*, No.4 (27), pp. 281-294.

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Völgyesi L. (2004) Conversion between Austrian and Hungarian map projection systems. Periodica Polytechnica Civ. Eng, Vol. 48, Nr. 1-2. pp. 73-88.

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