GPS AS THE DEVICE OF JUNCTION OF TRIANGULATION NETWORKS

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Abstract

Conversion between Hungarian and Austrian map projection systems is presented in this paper. The conversion may be performed in two steps: at first any kind of map projection systems should be transformed into WGS-84 ellipsoidal co-ordinates in one country, and then from WGS-84 ellipsoidal co-ordinates should be transformed into the desired system for the other country. An algorithm and a computer program has been developed to carry out this transformation.

1. Introduction

Map projection systems of large scale maps and their reference surfaces, as well their triangulation networks usually differ in different countries - moreover may be different inside the same state. A contiguous triangulation network can be considered as uniform according to map projections only if the whole network was adjusted at the same time. Geodetic coordinates of triangulation stations will be changed even if the network had been readjusted, e.g. with new conditions.

It is possible to make exact conversions between two map projection systems with closed mathematical formulae in cases only when both projection systems has the same reference surface and points of the same triangulation network coming from the same adjustment are represented in both projection systems. When any of the above mentioned requirements has not been met the conversion can be performed only using certain common points that have co-ordinates in both projection systems (Hazay, 1964; Varga 1981, 1982, 1986). In such case the accuracy of transformed co-ordinates depend on the reliability of triangulation networks and the position and number of selected common points. Slightly different co-ordinates will be resulted after the conversion process when other common points had been chosen. If there is no exact conversion using closed mathematical formulae between two map projection systems, the transformation can be performed only by Helmert's transformation or polynomials up to the maximum degree five (*Mannual for the Application of Unified National Projection,* 1975). Applying these methods we can eliminate the distortions of projection and the discrepancies of triangulation networks at the same process making a single plain transformation.

More precise and secure conversion can be made using the so-called *mixed method*. In this method the transformation can be performed in two steps: first the distortions of projection and than the discrepancies of triangulation networks can be eliminated. In the first step we suppose that the two map projection systems have the same reference surface and the same triangulation network, and we perform the computation by the *co-ordinate method* using closed mathematical formulae (Varga 1986). So in the first step we get approximate plane co-ordinates in the second projection system. Then in the second step we perform a transformation by polynomials using common points. The common points for determining the coefficients of this transformation polynomials should be that points, which have both the previously computed approximate values and the original plane co-ordinates in the second projection system. We can use transformation polynomials having lower degrees in the second step of transformation to eliminate the discrepancies of the different triangulation networks, against if we would make the conversion in only one step using series.

2. Conversion between Hungary and Austria

There are some difficulties in case of conversion between map projection system of neighboring countries - when the reference surfaces of the applied map projection systems are the same and the triangulation networks are connected on them. This is the case between the Hungarian and Austrian map projection systems that we have investigated. Conversion between Hungarian and Austrian map projection systems can't be computed by *co-ordinate method* using closed mathematical formulae because the position and orientation of reference surfaces are slightly different, and the triangulation networks had been adjusted one by one - although there is the Bessel's ellipsoid as a reference surface of projection systems which is applied in Hungary and Austria too, and there are some common points of different triangulation networks. So the conversion between the two countries can be performed only by transformation polynomials using common points.

Map projection systems of neighboring countries can be expanded only for a few ten kilometers range from the common border because common points can always be found only in this region. GPS is the most powerful tool for fixing common points anywhere, because determining of X, Y, Z spatial geocentric Cartesian, WGS-84, or UTM co-ordinates of points of triangulation network by GPS, we can create such system of common points which are very suitable for conversion of map projection system between the neighboring countries.

Such conversions between countries are necessary only if somebody want to use it's own special map projection system in the neighbor country and don't want to use e.g. the more simple UTM projection system. But in this case a projection system of a country can be expanded into a direction only within the pale of reason, not too far from the common border.

Having enough common points determined by GPS afford possibility to make computer program for the conversion between map projection systems of Hungary and Austria. So it is all the same, that we transform co-ordinates between map projection systems of Hungary and Austria with different reference surfaces (Bessel's ellipsoid in Austria, and Bessel's, Krassovky's or IUGG-67 ellipsoids in Hungary) and different meridian of origin (prime meridian of Ferro for Austria and prime meridian of Greenwich for Hungary).

3. Practical solution

The conversion logic between the different map projection systems can be overviewed on *Fig. 1*. Transformation paths – and their directions – between different systems are pictured by arrows. It can be seen that it is possible to convert between both WGS-84 ↔ Unified National Projection and WGS-84 ↔ Gauss-Krüger systems only through other intermediate systems. E.g. if a conversion between WGS and EOV systems is needed then WGS-84 co-ordinates first have to be converted into the new Gaussian sphere (NGS) and then into a so-called auxiliary system (AUX) and finally they should be converted from this AUX system into EOV co-ordinates; or e.g. if a conversion between GAK and WGS systems is needed then Gauss-Krüger co-ordinates first has to be converted to an auxiliary system (AUX) and then to the new Gaussian sphere (NGS) and finally they should be converted from the new Gaussian sphere to the WGS-84 ellipsoid.

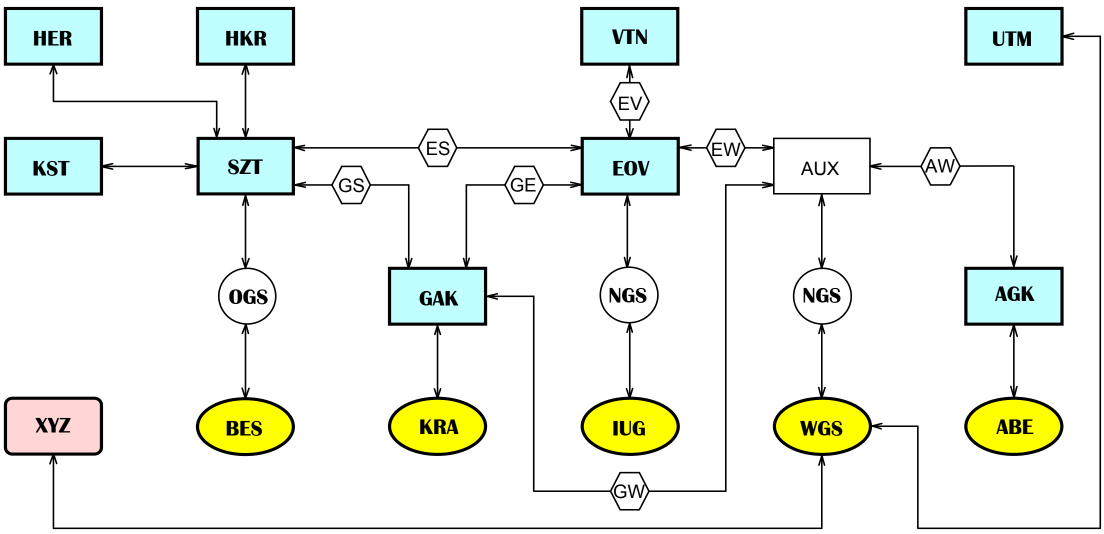

Fig. 1. Conversion logic between the different map projection systems

If any two systems in *Fig. 1* are connected through a hexagonal block then between these two systems only an approximately accurate conversion could be made by transformation polynomials. In *Fig. 1* the two-letter abbreviations in hexagonal blocks show which data files, containing transformation polynomials, have to be used to convert between the two neighboring systems. If any two systems in *Fig. 1* are connected by a continuous line then an exact conversion by the co-ordinate method, i.e. through closed mathematical formulae can be made.

Since it may cause problems even for experts to apply correct methods of conversion between a multitude of map projection systems so such a program package has been developed by which conversions can be made between Hungarian and Austrian map projection systems and their reference co-ordinates in all combination, the usage of which can cause no problem even for users having no deeper knowledge in map projections.

Conversion between co-ordinates

VTN = System without projection
BES = Hungarian Bessel's Ellipsoidal
SZT = Budapest Stereographic Projection
KST = Military Stereographic Projection
HER = North Cylindrical System
HKR = Middle Cylindrical System
ABE = Austrian Bessel's Ellipsoidal
AGK = Austrian Gauss-Krüger Projection
IUG = IUGG-67 Ellipsoidal
EOV = Unified National Projection
KRA = Krassovsky's Ellipsoidal
GAK = Hungarian Gauss-Krüger Projection
WGS = WGS-84 Ellipsoidal /GPS/
XYZ = Spatial Cartesian Geocentric /GPS/
UTM = Universal Transverse Mercator

are performed by the conversion program in the area of Hungary and Austria in 212 combinations as it is enlisted in *Table 1*. South cylindrical projection system (HER) and Budapest city stereographic projection (VST) are not to be found on the above list because the regions where these two Hungarian map projection systems are used, is not neighboring to Austria and using these two systems there is no practical need to make conversion between Hungary and Austria.

Table 1. Possible transformations between Hungary and Austria.

	VTN	BES	SZT	KST	HER	HKR	ABE	AGK	IUG	EOV	KRA	GAK	WGS	XYZ	UTM
VTN	-	×	×	×	×	×	×	×	×	×	×	×	×	×	×
BES	×	-	+	+	+	+	×	×	×	×	×	×	×	×	×
SZT	×	+	-	+	+	+	×	×	×	×	×	×	×	×	×
KST	×	+	+	-	+	+	×	×	×	×	×	×	×	×	×
HER	×	+	+	+	-	+	×	×	×	×	×	×	×	×	×
HKR	×	+	+	+	+	-	×	×	×	×	×	×	×	×	×
ABE	×	×	×	×	×	×	-	+	×	×	×	×	×	×	×
AGK	×	×	×	×	×	×	+	!+!	×	×	×	×	×	×	×
IUG	×	×	×	×	×	×	×	×	-	+	×	×	×	×	×
EOV	×	×	×	×	×	×	×	×	+	-	×	×	×	×	×
KRA	×	×	×	×	×	×	×	×	×	×	-	+	×	×	×
GAK	×	×	×	×	×	×	×	×	×	×	+	!+!	×	×	×
WGS	×	×	×	×	×	×	×	×	×	×	×	×	-	+	+
XYZ	×	×	×	×	×	×	×	×	×	×	×	×	+	-	+
UTM	×	×	×	×	×	×	×	×	×	×	×	×	+	+	!+!

This table conveys us information on the possibility and accuracy of conversions very simply.

Double lines in this table separate map projection systems belonging to different reference surfaces. (By reference surface the ellipsoid is meant, though the fact should be acknowledged that the approximating /Gaussian/ sphere serves also as a reference surface for those map projection systems where a double projection is applied and an intermediate sphere is the reference surface at the second step of the projection to get co-ordinates on a plane. Co-ordinates on this approximating sphere have no practical role for users.)

Plus " + " signs at the intersection fields of rows and columns indicate that an exact conversion between the two map projection system is possible using closed mathematical formulas found in reference works of (Hazay, 1964) and (Varga, 1981, 1986) for transformation. In this case the accuracy of transformed co-ordinates is the same as the accuracy of co-ordinates to be transformed.

Cross " Í " signs of this table indicate the impossibility of transformation between the two map projection systems with closed mathematical formulae and the conversion – according to rules found in (Mannual for …, 1975) is performed using polynomials as of a finite (maximum five) degree with limited accuracy (Völgyesi at all, 1994, 1996; Völgyesi, 1997).

Minus " – " signs in the table are reminders of the fact that an identical (transformation into itself) conversion has no meaning except of the Gauss-Krüger and UTM projection systems where the need of conversion between different zones frequently arises. Hence a " !+! " sign indicates that it is possible to make exact conversions between different zones of the Gauss-Krüger and UTM map projection systems.

3. Accuracy of conversions

It was mentioned previously that it is possible to convert through closed mathematical formulae between certain map projection systems. A conclusion could have been drawn as a result of our test computations that in these cases the accuracy of transformed plane co-ordinates is equal to the accuracy of initial co-ordinates (*1 mm* or *0.0001"*). These conversions are referred to in *Table 1* with " + " and " !+! " signs or these systems are connected by continuous lines (arrows) in *Fig. 1*.

In all other cases when the transformation path between any two systems passes through an hexagonal block (or blocks), the accuracy of transformed co-ordinates depends, on one side, how accurately the control networks of these systems fit into each other; and on the other side, how successful the determination of transformation polynomial coefficients was. It follows also from these facts that no matter how accurately these transformation polynomial coefficients were determined, if the triangulation networks of these two systems do not fit into each other accurately – since there were measurement, adjustment and other errors during their establishment – then certainly no conversion of unlimited accuracy can be performed (in other terms, only such an accurate conversion between two map projection systems is possible that the accuracy allowed by the determination errors or discrepancies of these control networks). This fact, of course does not mean that one should not be very careful when the method of transformation is selected or – when the polynomial method is applied – the coefficients are determined.

Our first tests aimed at the question to decide which one of the two methods: Helmert transformation or polynomial method is more advantageous to be used. We arrived at the result that although the Helmert transformation is computationally more simple its accuracy in the majority of cases does not even approximate the accuracy provided by the polynomial method. Since a simple programming can be a motive for only software "beginners" therefore we took our stand firmly on the side of the use of polynomial method.

When the polynomial method is chosen the next important question is to determine the optimal degree of the polynomial. By considering a simple way of reasoning one could arrive at the conclusion that the higher the degree of the polynomial the higher the accuracy of map projection conversions will be. On the contrary, it could be proved by our tests that the maximum accuracy was resulted by applying five degree polynomials. No matter whether the degree was decreased or increased, the accuracy of transformed co-ordinates was lessened alike (more considerably by decreasing, less considerably by increasing).

Coefficients of transformation polynomials based on co-ordinates of common points y_i, x_i and y_i', x_i' in systems *I* and *II*, respectively. Then y_i, x_i co-ordinates in system *I* are transformed into co-ordinates ty_i', tx_i' in system *II* by using these coefficients and finally the standard error characteristic to conversion,

$$\mu = \sqrt{\frac{\sum_{i=1}^{n}(ty_i'-y_i')^2 + \sum_{i=1}^{n}(tx_i'-x_i')^2}{n}} \tag{1}$$

will be determined.

For information it could be mentioned that for example, between the Budapest Stereographic and the EOV systems the standard error is *±0.252 m* from the expression (1) for the complete area of Hungary when 134 common points are used and the same figures are *±0.037 m* and *±0.217 m* between EOV and WGS-84, and EOV and Gauss-Krüger systems by using 34 and 50 common points respectively. Between Austrian Gauss-Krüger and WGS-84 systems the standard error is *±0.152 m* for the complete area of Austria when 57 common points are used to determine the coefficients of transformational polynomials.

Our experiences showed the fact that although the accuracy can somewhat be increased by increasing the number of common points within the polynomial method but the accuracy of conversion can not be increased beyond a certain limit even with this method since there is a difference between the two triangulation networks. In certain cases, however, an improvement could be gained when transformation polynomial coefficients are not determined for the complete area of the country but for only smaller region common points are given and transformation polynomial coefficient are determined. In such cases conversions, of course, must not be made outside the sub-area where the coefficients of transformation polynomials were determined, and the junction of these regions is not a simple problem.

4. Summary

Conversion between Hungarian and Austrian map projection systems is performed in two steps: first any kind of map projection systems should be transformed into WGS-84 ellipsoidal co-ordinates in one country, and then from WGS-84 ellipsoidal co-ordinates should be transformed into the desired system for the other country. A precise and secure conversion can be made using the so-called *mixed method:* first the distortions of projection, and than the discrepancies of triangulation networks can be eliminated performing a transformation by polynomials using common points. Using our method and software for the given common points,

the transformation between Austrian and Hungarian map projection systems can be performed with a few centimeters accuracy for a few ten kilometers range of common border.

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