INTERPOLATION OF DEFLECTION OF THE VERTICAL FROM HORIZONTAL GRADIENTS OF GRAVITY

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Zusammenfassung

Der Verfaseer skizziert eine weiterentwickelte Lösung der, auf der Basis der Schweregradientenwerte liegenden Interpolationsverfahren der Lotabweiohungen und macht die Ergebnisse eines zum Zwecke der praktischen Erprobung dieser Methode durchgeführten Versuche bekannt. Auf Grund der erhaltenen Ergebnisee ist festzustellen, daß sich die auf diesem Wege erhaltenen mittleren Fehler der Komponenten der interpolierten Lotabweichungen unter den gegebenen Bedingungen kleiner als ± 0.7 Winkelsekunden ergaben.

Zum Sch1uss weist der Verfasser darauf hin, daβ man durch die Anwendung der auf diese Art und Weise verdichteten Werte der Lotabweichungen erstens sehr ausführliche und genaue Geoiddarstellungen herstellen kann und daß man zum anderen wichtige Informationen über die Massenverteilung unter der Oberflache gewinnen kann.

Knowledge of the deflections of the vertical helps to solve two great problems: partly it yields important data for the precise determination of the geoid, partly gives information to geoscientists about the distribution of underground, covered masses. In both instances a very dense net of deflections of the vertical is necessary. Astrogeodetic determination of deflections of the vertical is extremely lengthy and expensive, therefore in practice a sparser net of astronomical stations is usually established and this astrogeodetic net is interpolated by different methods.

In this paper the interpolation of deflections of the vertical based on gravity gradient. observations by torsion balance ere discussed, because the extensive flat territories of Hungary have earlier been precisely surveyed by torsion balance observations for other purposes. The method can be useful also in other countries in similar cases.

Basis of the theory of potential, a simple relationship can be written for the change of the deflection components and the gravimetric gradients of torsion balance observations between two points.

Loránd EÖTVÖS had worked out the principle of the method and János RENNER developed it further. In the last decades Ivan MUELLER and John BADEKAS (USA) carried out successful researches for practical uses of the method. Applying their results, investigations were begun in the Department of Geodesy of the Technical University Budapest. In the practical solution interpolation of the deflection of the vertical is performed along a chain of adjacent triangles. The origin and the end point of the chains are astrogeodetic points, where the deflection values are known and all the points of the net are at the same time torsion balance observation stations as well. If all the equations which can be written between the stations are taken into account, there will be more equations than unknowns. The most probable values of the unknowns are determined by adjustment.

Adjustment can be carried out in two different ways: either indirectly, by the usual method (through setting up and solving the norna1 equations) or directly (with the orthonormalization method).

The orthonormalization method was applied in our computations beside its other very favourable properties, chiefly because of a high characteristic numerical stability. Further advantages of the methods against the known procedures are that the accuracies of the interpolated values are independent of the geometric structure of the nets and of the azimuth of the direction between the origin and the end point.

After reading the input data, the computer program produces with the use of appropriate relationships, the so-called enlarged coefficient matrix and the right side vector; furthermore orthogonalizes the enlarged coefficient matrix resulting directly the deflection component values ξ and η and their mean square errors for each point of the net.

Finally, with the use of the interpolated deflections for each net point, the geoid undulations are computed - from which an other program traces a geoid map.

Some remarks about the data of the experimental computations.

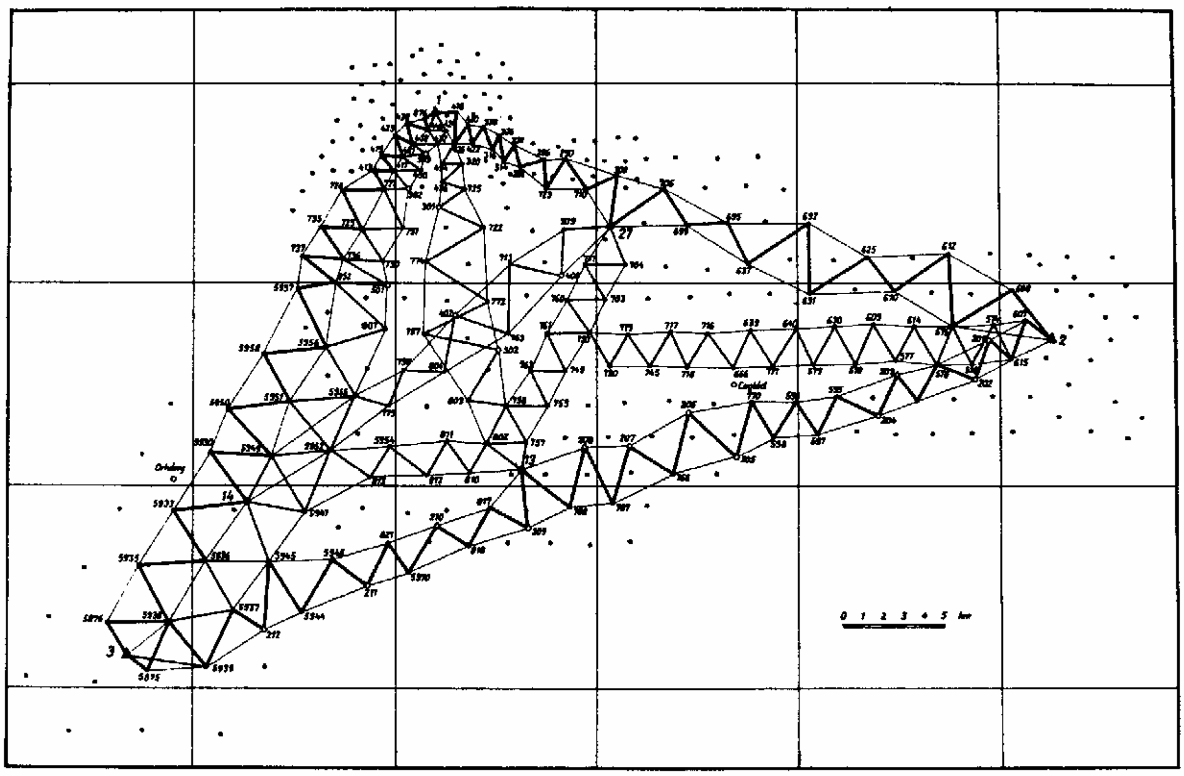

Fig. 1

For practically testing the method, the plane and hilly 1200 km^2 territory in Fig. 1 surveyed in detail by torsion balance observations was chosen, including three adjacent control points (1,2,3) with an average spacing of 40 km. For these control points the values of both the relative and gravimetric deflection components were available. In the shown area, deflections of the vertical were interpolated, based on torsion balance observations. For comparing or checking the results, the astrogravimetrically determined values for the three points (13,14,27) inside the area were used.

It appears from Fig. 1 that the torsion balance observation points were not uniformly distributed, observations were more concentrated in areas of increased gradients, in "more perturbed" areas. This has an importance in the development of interpolation nets, because the nets on territories of increased gradients are advisably established so that the change of the second potential derivatives of two adjacent points can be considered as linear. (This approximation was utilized to deduce the proper relationships).

Our experimental computations were carried out along the three principal interpolation lines, marked by a heavy line, the further six smaller chains were established to calculate geoid undulations.

Results of our computations are shown in Fig. 2. The arrows represent vectors to be considered either as horizontal force components or as direct deflection component values in an arbitrary system, defined by the initial values. (Former differ from latter only by multiplication by vector $\boldsymbol{g}$).

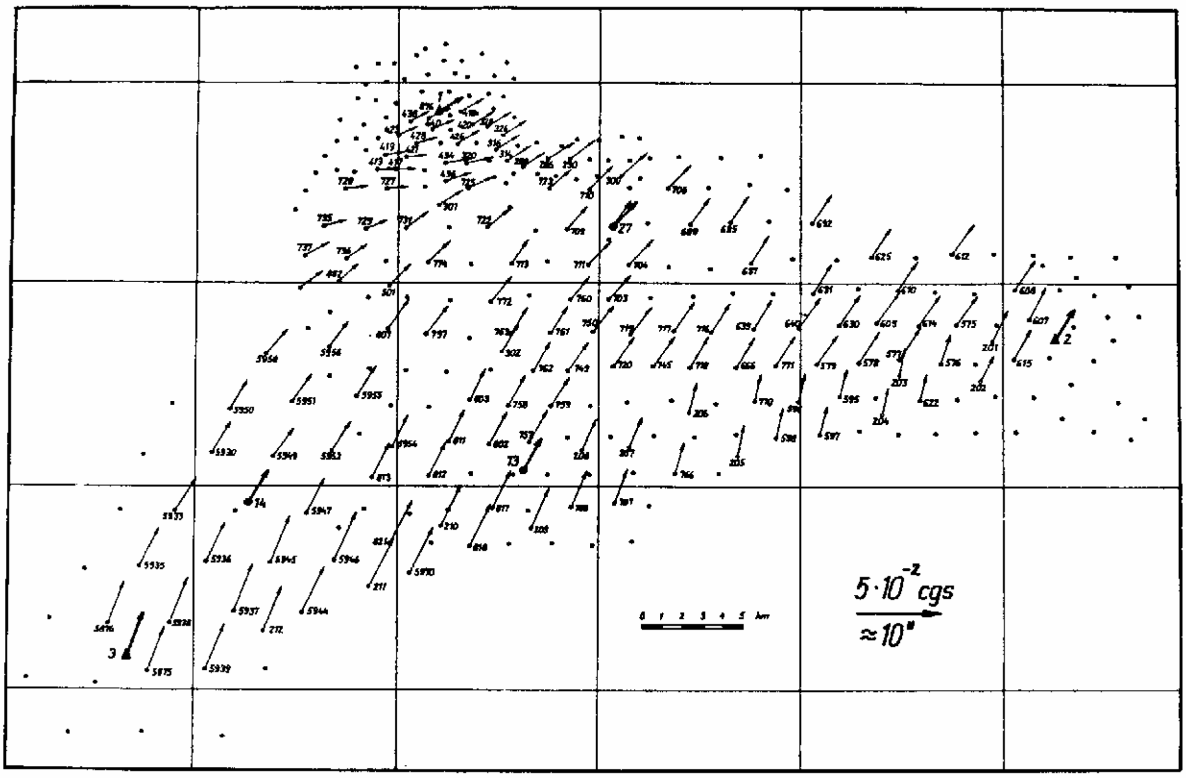

Fig. 2

In this way applying the adequate scale both the deflection values and the horizontal force components can be read off the figure.

The deflection values are marked by heavy lines (vectors) in the initial and the check points, the interpolated values are plotted with thin lines.

It can be stated that except for point 27 the given deflection values in the check points agree with the interpolated values within the graphic accuracy.

Also the mean square errors of the deflection components were computed far each net point. As expected, moving away from the extreme points of the chains, the mean square errors of interpolated values increased. In general the maximum of mean square errors were obtained in the middle parts of the nets, depending chiefly on the length of the interpolation line, the initial deflection components and the accuracy of the torsion balance observations. These mean square errors exceeded nowhere the value $\pm 0.7''$.

Finally some remarks about the applications.

As already mentioned, the deflections of the vertical help to solve two problems: partly they yield important data for the precise determination of the geoid, partly they give information about the distribution of subsurface masses.

One of the common problems of geophysics und theoretical geodesy is the precise determination of the geoid, the mathematical form of Earth. Today, based on up-to-date measurements, contour line maps of the main forms of the geoid for the whole Earth surface are available - these maps however do not contain the "fine structure" of the geoid.

On the territory in Fig. 1 computations were carried out using the previously interpolated deflection components to get a detailed geoid map by astronomical leveling.

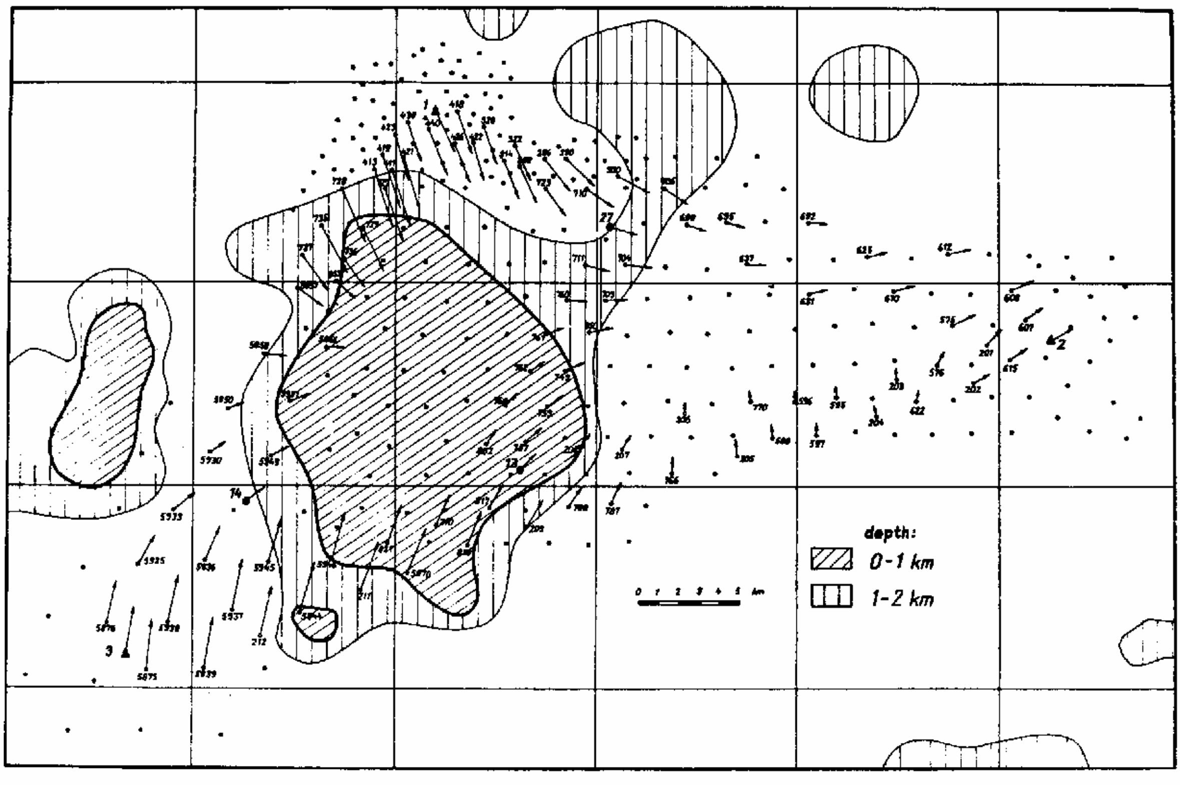

Fig. 3

It is characteristic for the accuracy of the determination that going along the chain length of about 115 km, misclosure of only 7 cm was obtained. This misclosure is characteristic not only of the accuracy of the geoid heights but is at the same time an excellent possibility to check the reliability of the interpolated ξ and η values.

Using interpolated deflection values based on torsion balance observations on flat areas, can be stated that highly reliable geoid maps are obtained very economically for territories where torsion balance observations are available. These are most suitable to study local details.

Computations were carried out also with deflection values determined by the gravimetric method. The vector diagrams of interpolated values obtained in this way are shown in fig. 3. The nearly central distribution of the vectors in this Figure offers to geoscientists abundant information. From the vector distribution around point marked 805, an effect of higher density must be supposed in the depth.

To check our results the obtained vector distribution was compared with the position of the subsurface magnetic active masses computed from the geomagnetic anomalies in the same area (Fig. 3).

A good accordance between the position of the magnetic bodies and distribution of the deflection is shown in the Figure.

References

[1] EÖTVÖS, R.: Gesammelte Arbeiten. Akadémia Kiadó, Budapest, 1953

[2] RENNER, J.: Deflection of the vertical, MTA Müsz. Tud. Oszt. Közl. V./1.-2. Budapest, 1952 (In Hungarian)

[3] RENNER, J.: Untersuchungen über Lotabweichungen. Acata Technica, XV. Budapeet, 1956

[4] RENNER, J. New investigations .... MTA Müsz. Tud. Oszt. Közl. XXI./1.-4. Budapest, 1957 (In Hungarian)

[5] BADEKAS, J.; MUELLER, I.I.: Interpolation of Deflections ... Reports of the Dep. of Geod. Sci. No. 98. The Ohio State University, 1967

[6] VÖLGYESI, L.: Interpolation Deflection ... Periodica Politechnica, Budapest, 1977 (under publication).

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