HUNGARIAN CONTRIBUTION TO THE RESEARCH IN GRAVIMETRY, GRAVITY FIELD MODELLING AND GEOID DETERMINATION (2015-2018) - IAG COMMISSION 2

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1 Introduction

This paper is made for the 27th International Union of Geodesy and Geophysics (IUGG) General Assembly 2019, Montreal, Canada as a quadrennial report of the Hungarian Contribution to the research in gravimetry, gravity field modeling and geoid determination (IAG Commission 2). In the past four years, we have conducted extensive and successful researches in the field of gravimetry, which are summarized below.

2 Hungarian Gravimetric Network

The Hungarian Gravimetric Network (MGH, Figure 1) is maintained by the Mining and Geological Survey of Hungary (the former Eötvös Loránd Geophysical Institute). According to the state in 2018, the MGH contains 24 absolute stations and 435 1st or 2nd order base points. The maintenance work includes the checking the status of base points as well as the substitution or installation of destroyed or new base points. Between 2015 and 2018, four absolute stations were installed, one base point was reinstalled, and one base point was newly installed. These stations were linked to the 3 nearest MGH base points by relative gravimetric measurements.

In order to improve the reliability and accuracy of the network, the gravity acceleration was redetermined (or newly determined in 4 cases) on 15 absolute stations between 2015 and 2018. The measurements were carried out by using the AXIS FG-5 No. 215 and 251 (H) absolute gravity meters operated by Vojtech Pálinkáš and Jakub Kostelecký (Výzkumný ústav geodetický, topografický a kartografický, v.v.i., Pecný, Czech Republic). Before the absolute measurements, vertical gravity gradient (VG) was determined on every station by LCR-G gravimeters, using a 3-level arrangement and at least 6 series of measurements.

Since the VG can deviate significantly from the normal value (−0.3086 mGal/m) in Hungary, vertical gradients were determined on further 26 base points between 2015 and 2018. Methodological examinations were also carried out by analyzing two-, three- and four-level VG measurements (Lévay and Koppán 2015).

Between 2017 and 2018 two newly installed INGA (Integrated Geodetic Point Network) base points were connected to the MGH network by comparative relative gravimetric measurements, the INGA stations were linked to the 3 nearest MGH base points.

In favour of the protection of MGH base points, recordings of base points were started into the Hungarian land and property registration system in 2018, according to the relevant rules of law.

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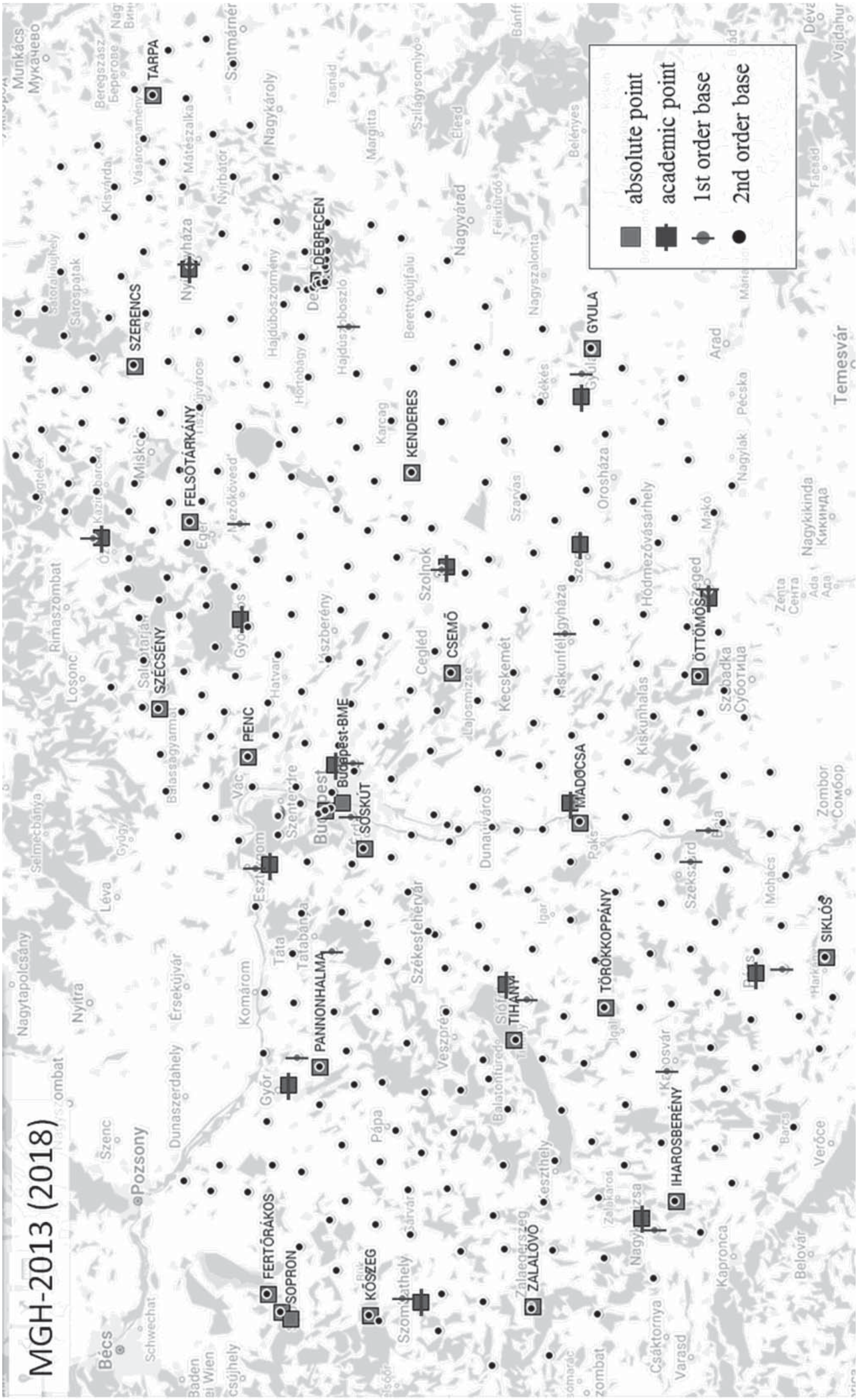

Figure 1. Hungarian Gravimetric Network in 2018

3 Remeasurement of the first gravity reference station of Hungary

The first gravity reference station of Hungary was established by Károly Oltay, professor and head of former Department of Geodesy, Technical University of Budapest (Ádám et al. 2018). The station was located in the pendulum hall and its gravity value (g = 980 852 mGal) was derived in 1915 from relative pendulum measurements with respect to the reference station at Geodetic Institute in Potsdam. The network of European gravity reference stations was adjusted by C. Morelli in the 1940's and he got g = 980 853 mGal for the station established by Oltay. This value only slightly differs from the one obtained by Finnish professor R.A. Hirvonen (g = 980 853.3 mGal), who used isostatic reduction in his calculations.

The pendulum hall was reconditioned in 2016, during which the reference station was identified and restored. Gravity was measured at the reference station (see Figure 2) between 26 and 27 May, 2016 with the FG5X (No.251) absolute gravity meter by staff members of the Czech Geodetic Observatory (Pecny, Ondrejov).

Vertical gravity gradient was also required to be measured at the station to reduce gravity measurements to the benchmark and to compare with the old g value. The measured vertical gravity gradient (−0.3091 mGal/m) served on the one hand to calculate g for the benchmark (allowing our reference station to be fit into the present operative gravity base network of Hungary (MGH-2013). On the other hand it facilitated calculating g for the benchmark established by Oltay for comparisons. When the systematic difference of −14 mGal between the reference system of Potsdam and the absolute system (or the value of −13.94 mGal based on local studies) is accounted for, the value obtained by us in 2016 (g=980 839 431.55 μGal) agrees well with the value obtained by Oltay (Ádám et al. 2018).

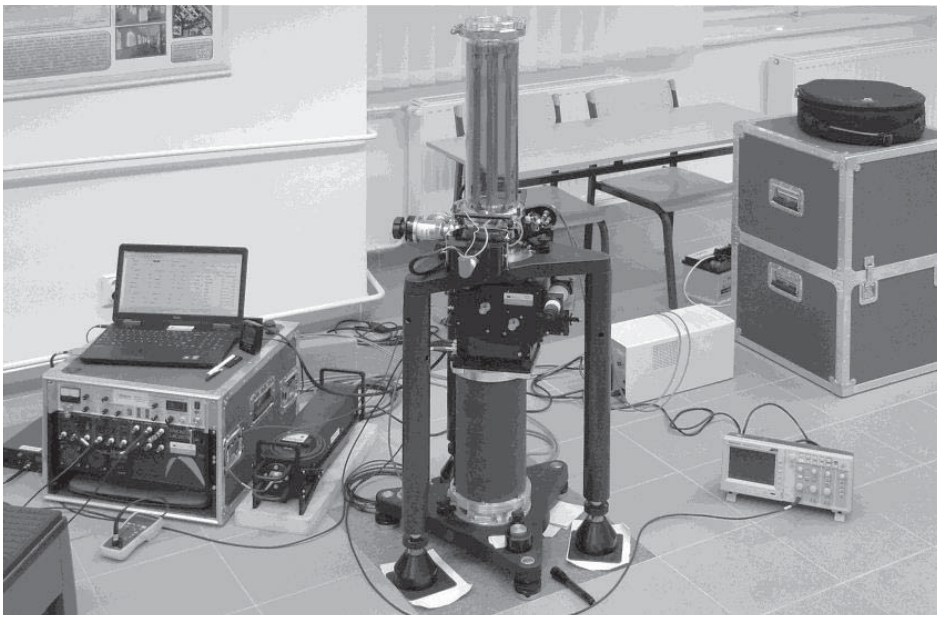

Figure 2. Measurement by the FG5X-251 absolute gravity meter at the Oltay-point

4 Torsion balance measurements

In the XX century, a large amount of torsion balance measurements have been carried out around the world. The measurements still provide a good opportunity to detect the lateral underground mass inhomogeneities and the geological fault structures using the so called edge effects in gravity gradients. Hitherto almost 60000 torsion balance measurements were made in Hungary mainly for geophysical purposes. Only the horizontal gradients were used for geophysical prospecting, the curvature gradients measured by torsion balance remained unused.

However, curvature gradients are very useful data in geodesy, using these gradients precise deflection of the verticals can be calculated by interpolation and using astronomical determination of the geoid the fine structure of the geoid can be derived. In our test area a geoid with few centimeters accuracy was determined based on the curvature data.

Based on the horizontal and the curvature gradients of gravity the full Eötvös tensor (including the vertical gradients) can be derived by the 3D inversion method.

Following the first big success in the 1910s and the second „Golden Age” in the 1950s torsion balance measurements for geological exploration have practically finished in Hungary by the end of the 1960s (Szabó 2016). After a long pause geodesy needed further measurements. Applying the new technical opportunities we reconstructed and modernized our older instruments, one of these instruments is an Auterbal balance of the Department of Geodesy and Surveying, BME (Budapest University of Technology); the other one is an improved type E54 instrument of ELGI (Eötvös Loránd Geophysical Institute) and the 3rd one is an Eötvös-Pekar torsion balance of the Geodetic and Geophysical Institute of the MTA Research Centre for Astronomy and Earth Sciences (see on Figure 3.). Field torsion balance measurements have been restarted in 2007 and additional torsion balance measurements have been made to study the linearity of gravity gradients (Völgyesi 2015, Völgyesi et al. 2015b). To reach the linearity of gravity gradients between the former torsion balance stations new measurements need to be made, the linearity mainly depends on the topography and the subsoil mass inhomogeneities.

In 2017 we began to reanalyse and remeasure of the classic Eötvös, Pekár and Fekete (EPF) experiment (Völgyesi et al. 2018, Péter et al. 2018), in which we found a systematic error. The objective of this experiment was to test what is now known as the weak equivalence principle (WEP), that the behavior of an object in a gravitational field is independent of its chemical composition. We've already started the actual measurements with a modified Eötvös-Pekar torsion balance.

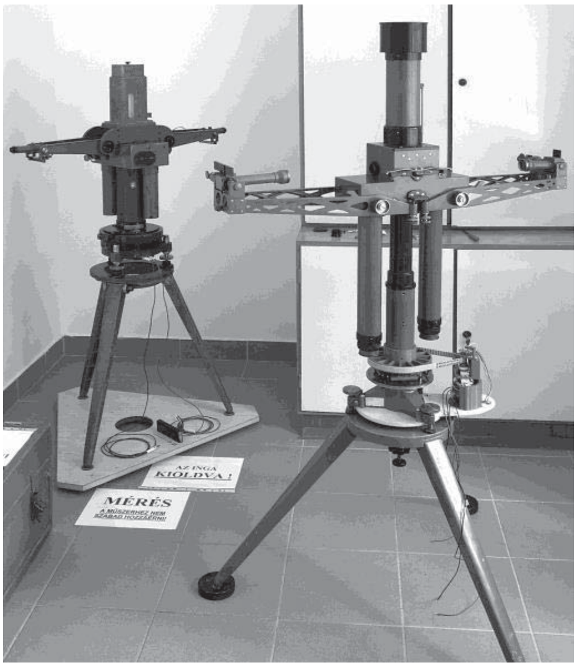

Figure 3. Renovated torsion balances at the Department of Geodesy and Surveying

5 Satellite gravity gradiometry

GRACE-borne temporal variations of the gravity field are proper tools for investigating mass redistribution processes on the seasonal and secular scales. It has been applied for investigating the water budget of the La Plata basin (Kiss and Földváry 2015a, Kiss and Földváry 2017b) resulting in an obvious change of periodicity along the river: while the upper sections are dominated by the annual periodicity of the water cycle, closer to the estuary due to the mixture with the runoffs of influents and subsurface waters, the periodicity vanishes.

GRACE monthly solutions are also used for monitoring present days ice mass variations at the polar caps. The most frequently used solution for Antarctica (Földváry et al. 2015a, 2015b) has been analyzed from error propagation perspectives. The efficiency of the ice mass balance investigations depend strongly on the reliability of the Glacial Isostatic Adjustment modelling (Földváry and Kiss 2016). All in all, it was found that over most area of Antarctica the accuracy of ice mass variations is not convincing, apart from the obvious melting process in West Antarctica and the mass accumulation in Enderby Land, no obvious mass variation can be detected (Kiss and Földváry 2017a).

By analysis of GRACE-derived mass time series, unexpected variations at multi-annual frequencies has been detected (Földváry and Kiss 2015). The locations of the multi-annual variations were identified with a combination of differently derived PSDs. The observed multi-annual variations were interpreted as consequence of large scale mass varying process, in most cases related to El Nino / La Nina events (Kiss and Földváry 2018).

6 Inversion reconstruction of 3D gravity potential function

Inversion reconstruction of 3D gravity potential based on gravity data measured by gravimeters, horizontal gravity gradients and curvature data measured by torsion balance and vertical gradients, including vertical deflection data have been obtained by our 3D solution. By applying this method the potential function – apart from an additive constant – and all the first and second derivatives of this potential function (elements of the full Eötvös tensor) can be determined not only at points of the region covered by measurements, but anywhere in the surroundings of these measurement points, using the coefficients of expansion in a series of a known set of basis function (Völgyesi et al. 2015a). The advantage of this method is that the solution can be performed by a significantly overdetermined inverse problem.

To verify the 3D inversion algorithm, test computations were made at the south part of Csepel Island in Hungary, where gravity, torsion balance and vertical gradient (VG) measurements have been performed, furthermore vertical deflection data were available from a new model. Torsion balance measurements were made here in 1950 and 30 new measurements were made in a denser net between the years 2006 and 2009, supplemented by gravity and vertical gradient observations. Deflections of the vertical have been determined for all points in the test area by the GGMPlus (Global Gravity Model, with plus indicating the leap in resolution over 149 previous 10 km resolution global gravity models) (Hirt 2013) model.

All the known horizontal gradients W_{zx}, W_{zy}, curvature data W_{xy}, W_{Δ}, vertical gradients W_{zz} and gravity values g were used as input data, but only a part of the known vertical deflection values were used as input data (as points for the training set) for the inversion; the remaining points (points of the validation set) were used for validating the computational results (15 points were considered for training, and 15 for the validation).

Different weights were applied for the input data: weights of the torsion balance measurements W_{zx}, W_{zy}, W_{xy}, W_{Δ} and the vertical deflection data were chosen to be 1 while the weights of gravity and VG measurements were chosen to be 10, as was the weight of the Laplace equation.

In our solution, comparing measured and computed data, we obtained practically the same horizontal gradients W_{zx}, W_{zy}, curvature data W_{xy}, W_{Δ}, vertical gradients W_{zz} and gravity values from the inversion as the input data of the measurements. In our study we focused mainly on how this inversion method can be applied to the determination of vertical deflections. From the 30 given vertical deflections 15 points were chosen as input data for the training set and are marked in Figure 4 by crosses; the remaining 15 points of the validation set as control points are marked by triangles. Differences $\Delta\xi = \xi^{(comp)} - \xi^{(GGMPlus)}$ and $\Delta\eta = \eta^{(comp)} - \eta^{(GGMPlus)}$ of the computed and given ξ , η components of the vertical deflections be seen in Figure 4. It can be seen that vertical deflections can be computed by this inversion method with $\pm 0.3 - 0.5''$ accuracy in our test area. Thus we have a very good possibility to compute vertical deflections with suitable accuracy based on the large amount and good quality of gravity and gravity gradient data in Hungary (Völgyesi et al. 2015a).

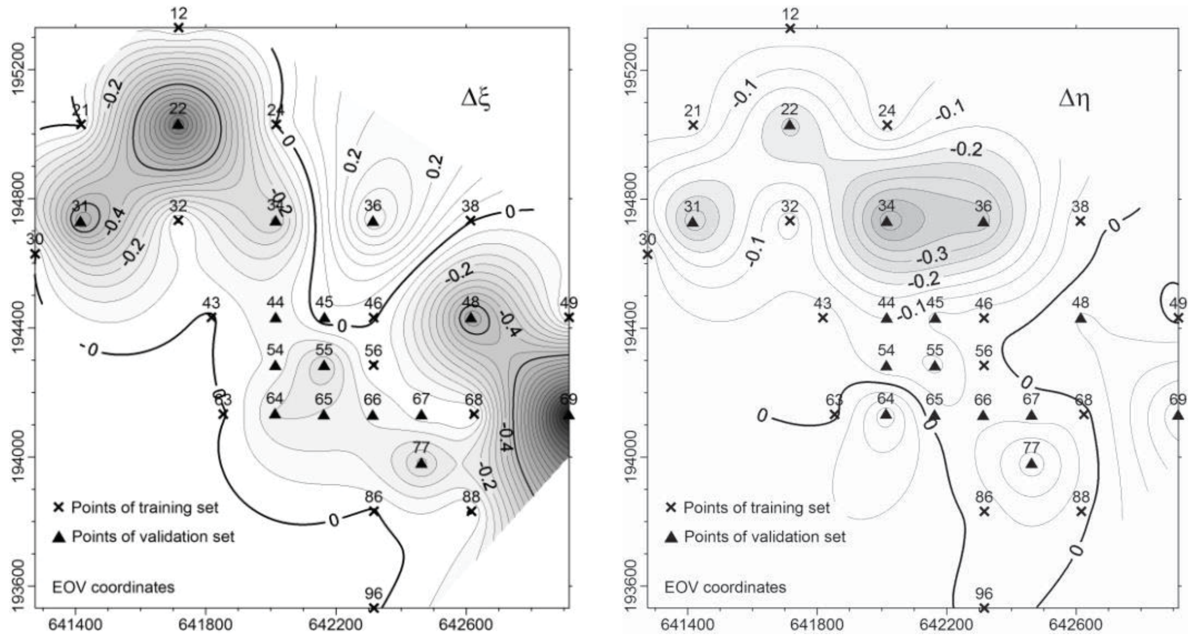

Figure 4. Differences between the computed vertical deflection components by inversion and the given ξ and η determined by the GGMPlus model within the test area. Points of training set are marked by crosses; points of validation set are marked by triangles. Contour interval is 0.05 arcsec

7 Correction of gravimetric geoid using symbolic regression

Unlike traditional linear and nonlinear regression methods that fit parameters to an expression (equation/relation) of a given form, symbolic regression (SR) searches both the parameters and the form of expression simultaneously. The proposed method could be of use to geodesy where regression analysis and functional approximation are often handled. For instance, SR could be used for gravimetric correction where they have been traditionally carried out using a wide variety of parametric and non-parametric surfaces such as polynomial models, spline interpolation, least squares collocation, kriging, combined least squares adjustments and thin plate spline (TPS) surface of solving the problem via finite elements method (Paláncz et al. 2015, see also http://library.wolfram.com/infocenter/articles/9444, 2019-10-01).

Some of these models are regression type global methods, e.g., linear parametric models and artificial neural network (ANN), while some of them are interpolating type local models as thin plate spline, support vector machines (SVM). Generally speaking local methods are more precise than global once on the training set, but their complexity is very high since they involve all of the measured data points in the model structure. In addition they proved to be less effective on the validation set since the overlearning affect the training set.

The result of this study indicates that SR can be a promising candidate to approximate functions and to find regression solutions in geodesy. However, the way to use SR method needs certain cautious, since there are many parameters which can be adjusted properly to get satisfactory result. According to our computational experiences, the best strategy is to let SR run until the structure of regression model has been found and then use standard nonlinear regression method to refine the parameters of the model. In addition finding a proper model structure depends strongly on the basic function-set used as building blocks of the SR. Therefore, one needs to select carefully these functions to find the proper solutions in an acceptable running time. Employing an intelligent guess for the initial population (i.e. Kreijzer expansion) is also indispensable. One should also keep in mind that the method is not a deterministic one therefore many repeated model generation runs were necessary. It is difficult to say which software package is the best choice, since the more effective package is the more sophisticated one and it is more difficult to adjust their parameters properly. So one should be cautious and not apply them blindly without prior study of their usage before (Paláncz et al. 2015, 2016).

8 Determinations of the vertical deflections

Our measurements and tests with QDaedalus predict revolutionary changes not only for the determination of deflections of the vertical (DOV) and local structure of the geoid but also for other fields of geodesy and surveying. This is corroborated by the fact that it is possible with the system in 30-60 minutes of measurement time only to get deflection of the vertical components which are accurate to 0.1 seconds of arc, or that it will be possible to determine geodetic astro-azimuths by luni-solar measurements with unprecedented accuracy (Völgyesi and Tóth 2015, 2016).

QDaedalus is an automated, computer-controlled astro-geodetic measurement system which has been developed at ETH Zurich, and is based on a transformed Leica Total Station that has been augmented with a GNSS receiver. This transformation mainly affects the optical part of the total station, and instead of its visual control it enables us to capture all CCD-assisted measurement data on the external controlling-processing computer hardware.

For the precise determination of deflection of the vertical, a very long sequence was measured at the point *Pistahegy*. The location of the measurements are shown in Figure 5, the WGS84 coordinates of point Pistahegy are, φ = 47˚24'53.8112" and λ=19˚08'17.8948". From the autumn of the year 2015 for three years we performed 180 measurements taken at 83 nights at this point in different seasons, in the most diverse meteorological, temperature and refraction conditions.

Before the measurements, the most important step is to calibrate the instrument. Accordingly, it is necessary to establish a connection between the readings on the horizontal and vertical circle of the total station and the readings in the coordinate system of the CCD (Charge-coupled device) sensor. By the previous procedure at night in field conditions, the calibration was rather cumbersome and did not meet the exact measurement accuracy. To solve this problem, using a collimator we have developed a new method and tool for calibrating more easily and more accurately. Studies were performed on the optimal calibration measurements and raster size; additionally, the temperature dependence of the measurements was also investigated (Völgyesi and Tóth 2018). Our experiences are useful in all cases when installing a CCD sensor for geodetic instruments.

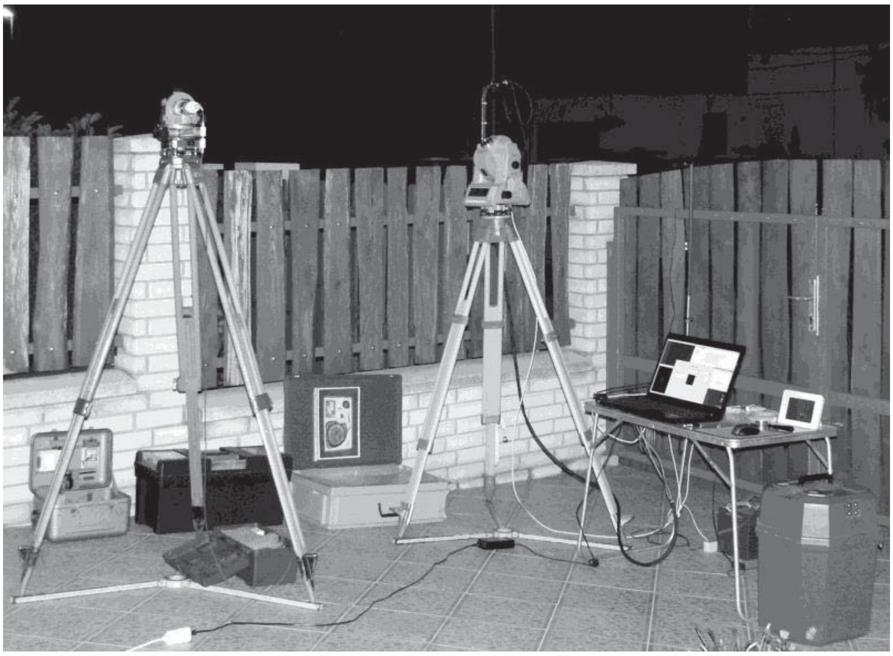

Figure 5. Night DOV measurement by QDaedalus system at Pistahegy point

We analyzed inversion residuals for the angles measured by the total station and found a horizontal angle bias for each new set of star observations. By incorporating the estimation of this bias in the least-squares Danish robust inversion procedure we reported 25% improvement in terms of standard deviation of the DOV components. On the other hand, inversion with Cauchy-Steiner weights by using bias estimation gave worst results by 5-10% in terms of standard deviation of DOV, and most frequent value procedure with Cauchy-Steiner weights is still the best on average by 19%. We also analyzed the measurement residual time series by using ensemble averaging. We found that stationarity no longer holds for the residuals obtained with horizontal circle bias estimation (Tóth and Völgyesi 2017). We have compared two robust and resistant methods for the inversion of QDaedalus zenith direction determination. M-inversion using Cauchy-Steiner weights yielded consistently better inversion results in terms of accuracy and correlation norm based on more than 70 independent measurement series. This is probably due to the higher statistical efficiency of M-inversion in case of a non-Gaussian distribution of residuals compared with the robust E-estimation with Danish method commonly used in geodetic data processing. By analyzing the accuracy of deflection of the vertical components we can state that 15-30 minutes of measurement time by QDaedalus is probably enough to reach DOV accuracy below 0.1″.

9 Advances in forward gravity field modelling

The research related to the application of different volume elements in forward gravitational modelling and its optimization regarding both the computation time and modelling accuracy has been continued in the Geodetic and Geophysical Institute, MTA CSFK by the support of NKFIH-OTKA project K101603. Beyond the rectangular prism the polyhedron, as a discrete volume element, can also be used to model the density distribution inside 3D geological structures. The calculation of the closed formulae given for the gravitational potential and its higher-order derivatives, however, needs twice more runtime than that of the rectangular prism computations. Although the more detailed the better principle is generally accepted it is basically true only for errorless data. As soon as errors are present any forward gravitational calculation from the model is only a possible realization of the true force field on the significance level determined by the errors. So if one really considers the reliability of input data used in the calculations then sometimes the "less" can be equivalent to the "more" in statistical sense. As a consequence the processing time of the related complex formulae can be significantly reduced by the optimization of the number of volume elements based on the accuracy estimates of the input data.

Detailed analysis of the modelling accuracy based on the law of error propagation and supported by numerical tests was presented and new algorithms were proposed (Benedek et al. 2018) to minimize the number of model elements defined both in local and in global coordinate systems. Common gravity field modelling programs generate optimized models for every computation points (dynamic approach), whereas the static approach provides only one optimized model for all. Based on the static approach two different algorithms were developed. The grid-based algorithm starts with the maximum resolution polyhedral model defined by 3-3 points of each grid cell and generates a new polyhedral surface defined by points selected from the grid. The other algorithm is more general; it works also for irregularly distributed data (scattered points) connected by triangulation. Beyond the description of the optimization schemes some applications of these algorithms in regional and local gravity field modelling were presented too. The efficiency of the static approaches provided even more than 90% reduction in computation time in favourable situation without the significant loss of reliability of the calculated gravity field parameters.

10 Advances in gravimetry

In the framework of NKFIH-OTKA project K101603 (2012 – 2017) a new absolute calibration line was established in the vicinity of Sopron (Muck, Sopronbánfalva Geodynamical Observatory, and Fertőrákos Cave Theatre) with a maximum height- and gravity difference of 369.8 m and 80.149 mGal, respectively. The absolute g values were determined by Vojtech Pálinkáš and Jakub Kostelecký (Výzkumný ústav geodetický, topografický a kartografický, v.v.i., Pecný, Czech Republic) with an average accuracy of ±0.3 μGal. All the points can be easily reached by car and located in sheltered places. The stations can also be used to provide reference for the monitoring of the g variations in the tectonically active area of the Mur-Mürz fault system.

The team of NKFIH-OTKA project K101603 investigated also the characteristics and the metrological limits of the absolute calibration of spring type gravimeters using a cylindrical test mass moved vertically around the gravimeter by a lifting device operated in the Mátyáshegy Observatory (Koppán et al. 2016, 2017, Kis et al. 2018). The movement of the 3200 kg stainless steel mass generates a sinusoid-like calibrating signal having a peak-to-peak amplitude of 1102 nm/s^2. The careful determination of the geometrical and physical parameters of the test mass combined with the analytical modeling of its gravitational effect and the related uncertainties provides an accuracy of 3 nm/s^2 in absolute sense. The overall accuracy, however, is influenced by several environmental and instrumental factors which are investigated in detail. The conclusions are based on more than 400 experiments with 5 LCR G instruments. As a unique case a Scintrex CG-5 instrument was also involved in the tests what is probably the first attempt to the absolute calibration of this type of gravimeters ever done.

11 Updated Hungarian gravity field model with spherical radial base functions

GOCE has provided satellite-only global gravity field models with unprecedented spatial resolution (Földváry et al. 2015c). Updated gravity field solutions for Hungary were determined using the latest DIR R05 GOCE gravity field model. The solution methodology was least-squares gravity field parameter estimation using Spherical Radial Base Functions (SRBF). Regional datasets include deflections of the vertical (DOV), gravity anomalies, surface gravity gradients and quasigeoid heights by GPS/levelling (Tóth and Földváry 2015a, 2015b, 2015c).

Regional gravity field modeling in Hungary has a long history dating back to 1918, when first in the world a disturbing potential isoline map of Arad and its surroundings, based on astrogeodetic and torsion balance data was published by D. Pekár (Biró et al. 2013). Astrogeodetic geoid determinations starting from 1950 were followed by modern gravimetric geoid determinations at various institutions (Biró et al. 2013). The application of Least Squares Collocation (LSC) to include heterogeneous data in gravity field estimation resulted in the gravimetric-astrogeodetic-gradiometric solution HGTUB2007 (Tóth 2008) and an EGM2008-based astrogeodetic-gravimetric combined quasigeoid model (Tóth and Szűcs 2011).

Gravity field modelling by SRBFs has several merits:

- individual data-dependent weights can be introduced
- it is easy to incorporate different gravity field functionals (e.g. DOV or gravity gradients)
- computational burden is reduced with respect to LSC, where the number of unknowns equals the number of data
- rigorous parameter estimation and prediction with full covariance information is possible

The functional model of SRBF parameter estimation is

$$F(\mathbf{r}_i) + e(\mathbf{r}_i) = \sum_{k=1}^{k} x_k B^F(\mathbf{r}_i, \mathbf{r}_k) \tag{1}$$

Here F and e denote a measured gravity field functional and its error at point $\mathbf{r}_i$, $B^F(\mathbf{r}_i, \mathbf{r}_k)$ is the SRBF placed at point $\mathbf{r}_k$, k is the number of SRBFs and x_k are the unknown parameters. This equation can be written for the estimation of parameter vector $\boldsymbol{x}$ as a linear equation system with design matrix **A** and data vector **y** as

$$\mathbf{y} + \mathbf{e} = \mathbf{A}\mathbf{x} \tag{2}$$

The shape of the band-limited SRBF is defined by its coefficients b_n in the Legendre series

$$B(\mathbf{r}_i, \mathbf{r}_k) = \sum_{n=N_{\min}}^{N_{\max}} (2n+1) \left(\frac{\mathbf{r}_k}{\mathbf{r}_i} \right)^{n+1} b_n P_n(\hat{\mathbf{r}}_i, \hat{\mathbf{r}}_k) \tag{3}$$

where $P_n(\hat{\mathbf{r}}_i, \hat{\mathbf{r}}_k) = P_n(t_{ik})$ are Legendre polynomials of degree n, $t_{ik} = \hat{\mathbf{r}}_i, \hat{\mathbf{r}}_k$ is the cosine of the angle between $\hat{\mathbf{r}}_i, \hat{\mathbf{r}}_k$ unit vectors. From this expansion $B^F(\mathbf{r}_i, \mathbf{r}_k)$ can be obtained by applying the

particular differential operator D belonging to functional F on the SRBF $B(\mathbf{r}_i, \mathbf{r}_k)$.
The shape of SRBF depends on the chosen bandwidth and coefficients b_n. We have chosen the most simple band-limited Shannon radial basis functions (Figure 6).

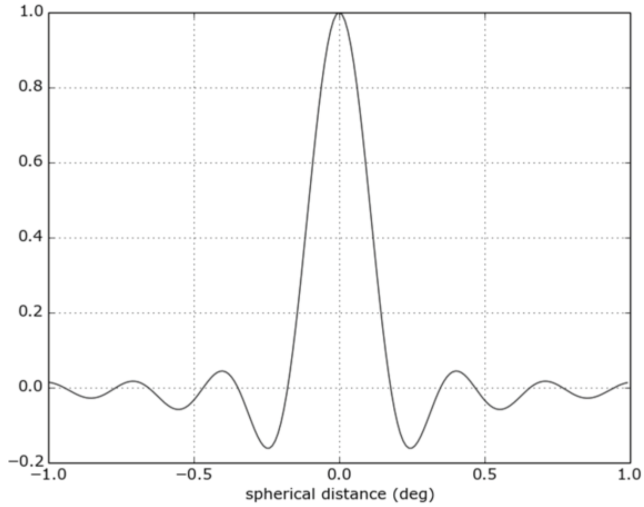

Figure 6. Shannon Radial Basis Function (bandwidth = 200-1200)

The positions of SRBF, i.e. points $\mathbf{r}_k$ were fixed to that of a Reuter grid. The distance between adjacent Reuter grid points is defined by the control parameter c. This parameter was chosen equal to the bandwidth of the base functions N_{max}.

Gravity field recovery from observed gravity data is an inverse problem. Instability of the inverse operator must be fixed by proper regularization to obtain a physically meaningful solution. For weighting and regularization the method of Variance Component Estimation (VCE) was used.

After the solution $\hat{\mathbf{x}}$ was estimated, the covariance matrix $\mathbf{C}_{\hat{\mathbf{x}}}$ of the parameters was determined from the law of covariance propagation as

$$\mathbf{C}_{\hat{\mathbf{x}}} = \sum_i \frac{1}{\sigma_i^2} \mathbf{N}^{-1} \mathbf{N}_i \mathbf{N}^{-1} \tag{4}$$

Prediction (synthesis) of gravity field functionals at non-measured points is possible by introducing $\hat{\mathbf{x}}$ into an equation similar to Eq. 2, $\mathbf{y}_S = \mathbf{A}_S \hat{\mathbf{x}}_S$. The full covariance matrix of predicted $\hat{\mathbf{y}}_S$ functionals is again computed by covariance propagation

$$\mathbf{C}_{\hat{y}_S} = \mathbf{A}_S \mathbf{C}_{\hat{x}} \mathbf{A}_S^T \tag{5}$$

The following 11 terrestrial datasets have been prepared for regional gravity field determination in the GRS80 system.

- Δg free-air anomalies of the MGH-50 gravity network (Renner and Szilárd 1959) (509 points)
- ξ, η astrogeodetic deflections of the vertical of the first-order control network (138 points)
- ζ quasigeoid undulations at OGPSH GPS control network:
  - ζ at points of the old Bendefy levelling network (87 points)
  - ζ at points of the EOMA levelling network (149 points)
  - ζ at points of the EOMA levelling network with recently levelled heights (97 points)
- v_{xz}, v_{yz} horizontal surface gravity gradients measured by torsion balance (37610 points)
- v_{yy-xx}, $2v_{xy}$ surface curvature gravity gradients measured by torsion balance (37272 points)

The last ζ dataset contains quasigeoid heights computed from a geopotential model (GPM) outside Hungary at points of a dense Reuter grid. The total number of data included in the solution was 154477. We have used three different GPMs. The GOCE DIR R05 model has been combined with the EGM20008 model according to optimal spectral content and has been evaluated in comparison with the EGM2008 and EIGEN-6C4 models to assess the performance of our regional gravity field solution.

All datasets have been reduced to contain information only between the chosen N_{min} and N_{max} SH degrees. First, frequencies above SH degree 2160 have been removed by the ERTM2160 model (Hirt et al 2013). Second, contributions of the GPM up to N_{min} and from N_{max} to 2160 have also been removed. Best data reduction is achieved by combined GOCE fifth generation models in bandwidth 200-1200. The bandwidth of the SRBFs, i.e. SH degrees N_{min} and N_{max} have to be chosen in accordance with extension of the data area and data density, respectively. We have chosen N_{min}=200 and N_{max}=1200.

The solution, i.e. the estimated SRBF parameters is shown in Figure 7 for the combined GOCE DIR R05 – EGM2008 GPM based model.

In Table 1 final variance factors as well as signal-to-noise ratios are shown for selected datasets. VCE iteration converges to these final variance factors σ_i^2 which indicate final relative weights of each dataset. Large variance factors show that those data were weighted down relative to others.

Another measure is the signal-to-noise ratio, SNR in dB, defined as the logarithmic ratio of the norm of the signal $\mathbf{y} = \mathbf{A}\hat{\mathbf{x}}$ with respect to norm of residuals $\hat{\mathbf{e}}$:

$$SNR = 10 \log_{10} \frac{\|\mathbf{A}\hat{\mathbf{x}}\|}{\|\hat{\mathbf{e}}\|} \tag{6}$$

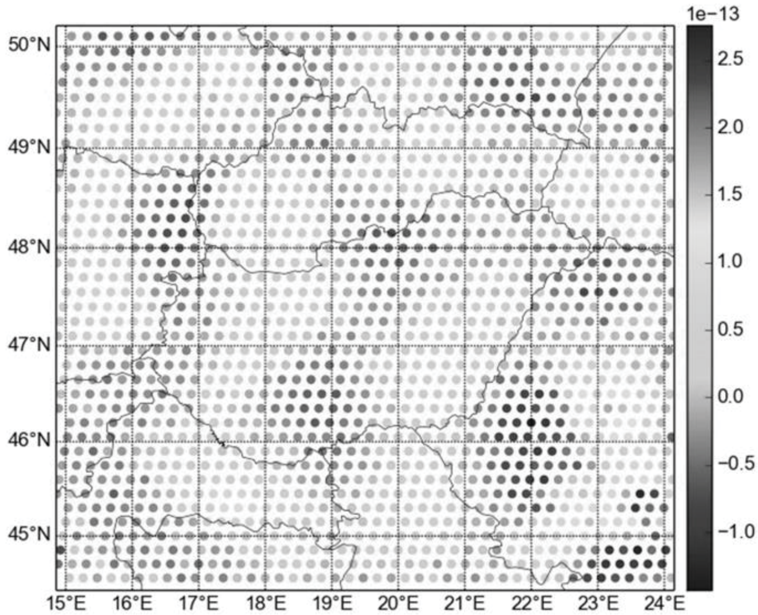

Figure 7. Estimated SRBF parameters of the geopotential according to Eq. 1. The parameters are normalized with the zero-degree term of the geopotential and hence unitless

Table 1. Variance factors σ_i and SNR ratios (dB) of selected datasets for GOCE DIR R05 based solution

dataset / GPM		ξ [“]	v_{xz} [E]	$\zeta_{Bend.}$	ζ_{EOMA}	ζ_{EOMARL}	$\zeta_{GPM\ outside}$	Δg
GOCE DIR R05 + EGM2008	σ_i	1.83	14.1	0.22	0.26	0.25	0.42	10.33
	SNR	1.91	-2.88	4.05	1.86	3.65	7.92	2.57

The relative accuracy of parameters can be defined as the ratio of parameter standard deviation and standard deviation of the solution vector. These relative accuracies are plotted on Figure 8 for the combined model.

The solution vector $\hat{\mathbf{x}}$ is used to predict quasigeoid heights and their covariances according to Eq. 5 on a regular grid. These predictions can be seen on Figure 9 and 10, respectively.

Finally to assess the accuracy of the solutions we predicted quasigeoid heights inside Hungary on a Reuter grid and compared the predictions with those of the GPMs in the frequency band 200-1200.

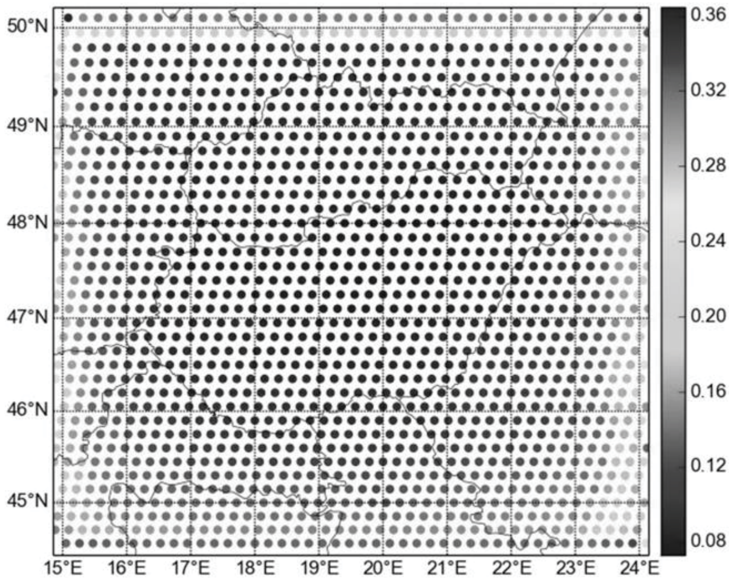

Figure 8. Relative accuracy of SRBF parameters

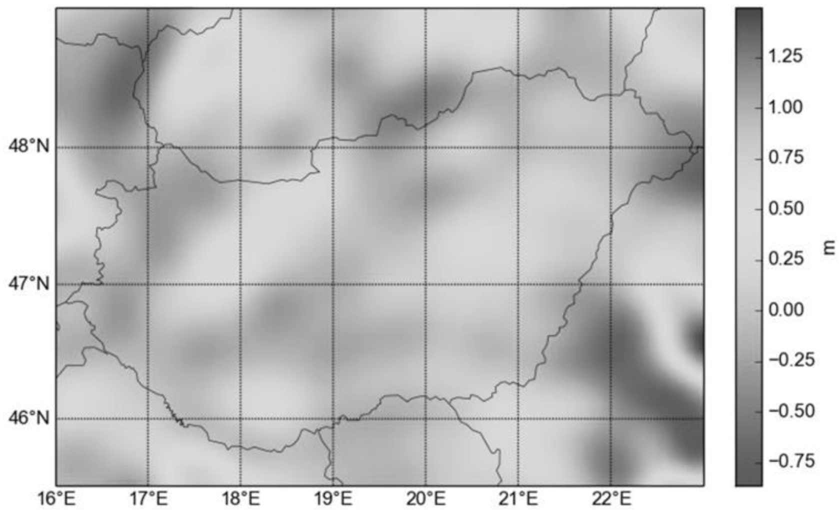

Figure 9. Estimated residual ζ quasigeoid in the SH band 200-1200

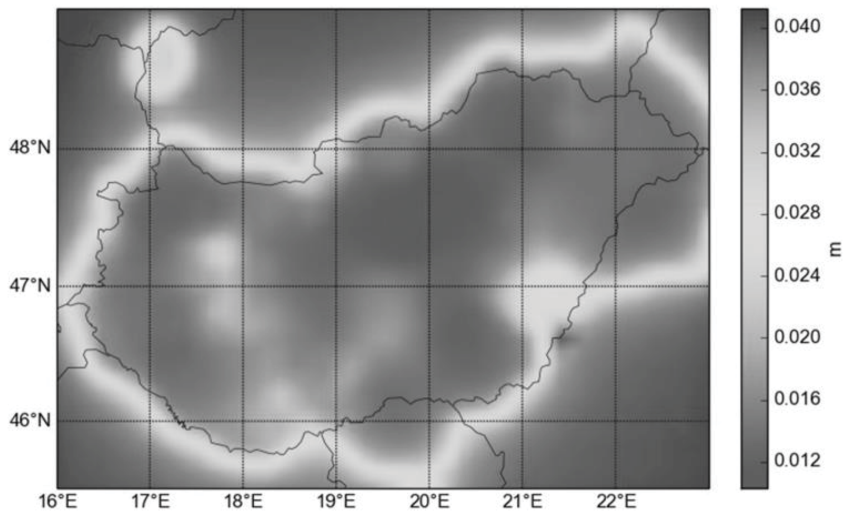

Figure 10. Estimated std error of residual ζ quasigeoid in the SH band 200-1200

Table 2 shows statistics of this comparison for the three tested geopotential models. These statistics show slightly better standard deviation of the residuals for the EGM2008 and EIGEN-6C4 models in the selected bandwidth of 200-1200, where residuals are defined as the differences of predicted SRBF and GPM quasigeoid heights. The mean of the residuals, however is the best for EIGEN-6C4 and our combined GOCE – EGM2008 model.

Table 2. Statistics of predicted quasigeoid height residuals at 1024 points of the three different SRBF gravity field models

GPM used	min	max	mean	std
EGM2008	-0.187	0.227	0.015	**0.051**
EIGEN-6C4	-0.189	0.224	**0.013**	**0.051**
GOCE DIR R05 and EGM2008	-0.206	0.219	**0.013**	0.056

The approach of regional gravity field modelling by SRBFs works quite well and provides reasonable results. One important merit of this technique is that data of different kind and heterogeneous distribution can easily be combined in a rigorous estimation procedure of gravity field parameters and their errors. We demonstrated this aspect by using DOV, gravity, gravity gradient and GPS/levelling data in combination with different geopotential models.

Since there is a direct connection one can easily compute SH coefficients from the estimated SRBF coefficients. This enables straightforward estimation of signal and error degree variances of regional datasets, which is another benefit of the regional gravity field modelling by SRBFs.

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