Modelling gravity gradient variation due to water mass fluctuations

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Abstract. Two case studies were considered where gravity field changes were detected resulting from water mass variations. The first case is an urban water reservoir where the maximum daily change of water is 40000 m^3. The 3D model of the water mass allowed us to build an accurate polyhedral model of the variation of mass changes of the water. This mass density variation model made it possible to compute and compare variations of various gravitational field functionals. Gravity and full gravity gradient tensor changes were computed on a regular grid for the model area. Gravity changes were also compared with actual gravity field measurements made with two LCR gravimeters. The measured gravity change was nearly 30 μGal relative to 5000 m^3 water mass. A good agreement was found between the computed and measured changes. The second case is the water level fluctuations of the Danube River in Budapest during the great flood in 2002. In this case also modelling and measurements were compared. We found the gravitational change to be very sensitive to the actual distance of the point from the river bank.

Keywords. Water mass variations, changing of gravity anomalies, changing of full gravity gradient tensor

1 The gravitational effect of water level fluctuations of a water reservoir

The gravity effect of the daily water level fluctuation of the Gellért Hill reservoir of Budapest, having 80000 m^3 capacities, was studied by relative gravity measurements, and model calculations were carried out to verify the measurements. Modelling sketch of this water reservoir can be seen on Figure 1. The underground water reservoir consists of two basins, the concrete ceiling is covered by 3−4 m of gravel and soil. One metre of water level change in either of the two basins corresponds to 5000 m^3 of water mass.

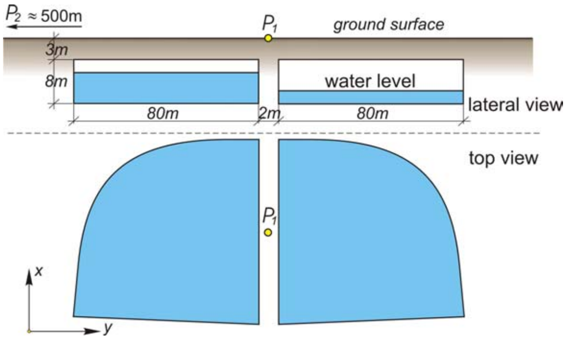

Fig. 1 Modelling sketch of the Gellért Hill water reservoir

For the gravity measurements one of the gravity stations was located above the reservoir at point P_1, the other station P_2 was at a distance of 500 m, where the water level change of the reservoir had already no gravitational effect. The observations were repeated several times with two gravimeters in the morning and in the afternoon. The volumes of water at the time of measurements were computed using the dimensions of the reservoir and the water level registration of the Metropolitan Waterworks.

Table 1. Gravimeter measurements above the Gellért Hill water reservoir, measured by G.Csapó (Csapó et al, 2003), ($x = \delta\Delta g / \Delta H$)

day/part		Gravimeter LCR 1919G					Gravimeter LCR 963G				
		Δg	δΔg	H	ΔH	x	Δg	δΔg	H	ΔH	x
1.	a.m.	477	126	7.35	4.82	26.1	475	116	7.11	4.59	25.5
	p.m.	351		2.53			359		2.52		
2.	a.m.	468	92	7.29	3.71	24.8	475	122	6.94	3.55	34.4
	p.m.	376		3.58			353		3.39		
3.	a.m.	484	102	7.42	3.60	28.3	477	93	7.22	3.91	23.8
	p.m.	382		3.82			384		3.31		
mean						26.4					27.9
average		27.2 μGal/m									

The observed Δg values are presented in Table 1, where Δg means the observed relative gravity in μGal (1 μGal = 10^{-8}m/s^2) for the given gravimeter and H means the height (in m) of water in the reservoir. The change per metre was calculated from the actual $\delta\Delta g$ and ΔH data, where $\delta\Delta g$ means the gravity variation due to the daily water level change. The result of these measurements provided a value of 27.2 μGal/m, which means that e.g. 5 m water level change (corresponds to 25000 m^3 of water mass) causes 0.136 mGal change of gravity at point P_1.

These measurements were controlled by theoretical computations by the software Mod3D written by I. Cerovský (Cerovský et al, 2004). Mod3D is an interactive 3D geophysical gravity and magnetic modelling software, it has been developed to create 3D geophysical models in a user friendly interactive environment. Mod3D computes anomalous gravitational field components $\mathbf{g}\,(g_x, g_y, g_z)$ and the full gravitational tensor elements W_{xx}, W_{yy}, W_{xy}, W_{zx}, W_{zy}, W_{zz}. Anomalous gravitational fields are computed using formulae for polyhedral bodies. The application of polyhedron volume element can provide a more realistic geometrical description of boundary surface of a geological body (e.g. topographic surface, without height jumps) than the description made by rectangular prism (parallelepiped) models. Using the polyhedron density model the generated second derivatives of the gravitational potential are more smooth and realistic function than the ones provided by prism model (Benedek, 2002). If the calculation level is near to the surface of the gravitational source, the accuracy of related quantities of gravitation can be increased by a detailed description of this surface in the vicinity of calculation.

Model computations were carried out for the right side basin on Figure 1 in case of thickness 5m (25000 m^3) water mass. The result of this computation provided a value of 0.15 mGal at point P_1. The difference between measured (0.136 mGal) and computed (0.15 mGal) values may be due to the uncertainty in the relative position of the observation point and the reservoir.

Computations were performed not only for the point P_1, but for a few hundred meters surroundings of the water reservoir. In Figure 2 changes of gravity in the surroundings of water reservoir can be seen. Maximum gravity changes are exactly above the edge of water mass, and the change decreases strongly in the function of distance from the water reservoir.

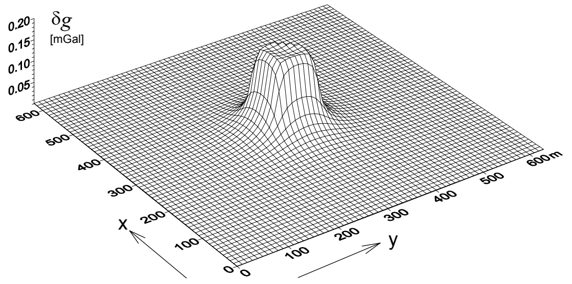

Fig. 2 Changes of gravity in the surroundings of water reservoir for the case of 50000 m^3 water mass.

At the same time the elements of full gravitational tensor were computed too for this water mass. The variations of Eötvös tensor elements W_{xx}, W_{yy}, W_{Δ}, W_{xy}, W_{zx}, W_{zy}, W_{zz} can be seen in Figures 3, 4, 5, 6, 7, 8 and 9 respectively in Eötvös unit (1 Eötvös Unit = 1E = $10^{-9}s^{-2}$).

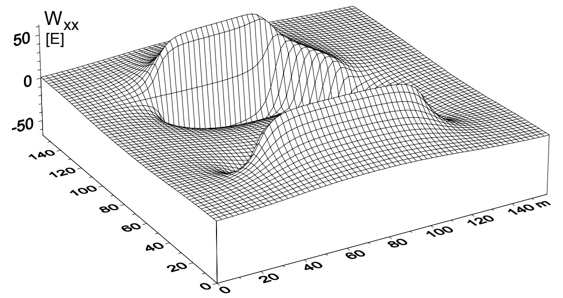

Fig. 3 Changes of gravity gradient W_{xx}

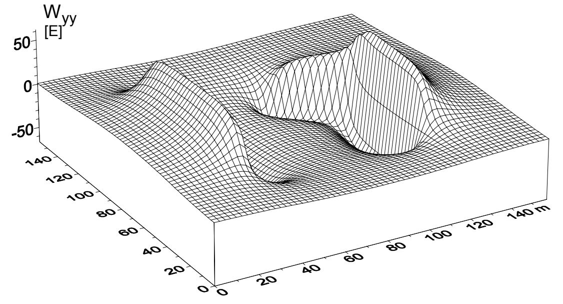

Fig. 4 Changes of gravity gradient W_{yy}

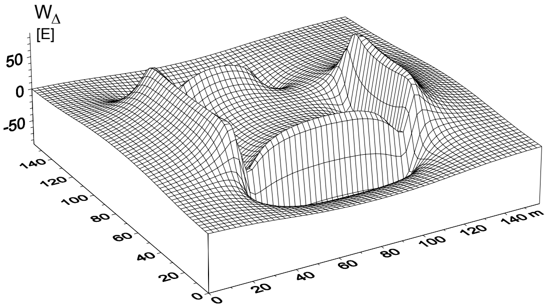

Fig. 5 Changes of curvature data $W_\Delta = W_{yy} - W_{xx}$

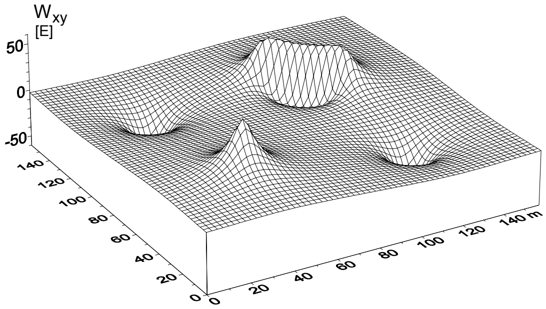

Fig. 6 Changes of curvature data W_{xy}

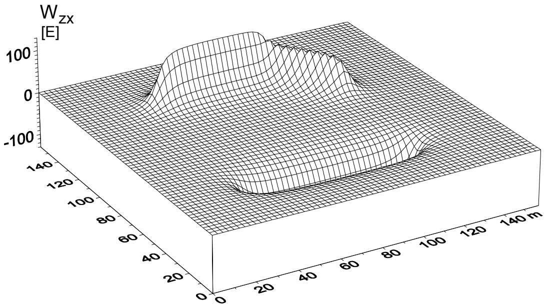

Fig. 7 Changes of horizontal gravity gradient W_{zx}

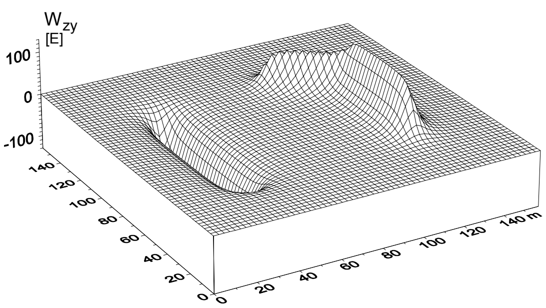

Fig. 8 Changes of horizontal gravity gradient W_{zy}

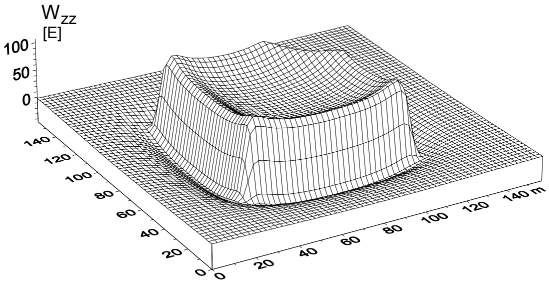

Fig. 9 Changes of vertical gravity gradient W_{zz}

As it can be seen on Figures 3, 4, 5 and 6 the changes of curvature data are between -50 and +50 E at the height of 4 m above the water mass. The biggest changes of W_{xx}, W_{yy}, W_{xy} are exactly above the edge of water mass, while in case of W_Δ at the corner points. The changes decrease strongly in the function of distance from the water mass. The changes of horizontal gradients of gravity W_{zx} and W_{zy} are two times bigger than the changes of curvature data. As it can be seen on Figures 7 and 8 the maximum changes are between -100 and +100 E above the edge of water mass. The changing of vertical gradient is more than 100 E above the water mass, which is about 3% of the normal value.

2 The gravitational effect of water level fluctuations of the Danube River

In August of 2002 there was a great flood of Danube River and gravity measurements were carried out to study the effect of water level fluctuations of the river. One station was located on the embankment at Szabadság bridge in Budapest, the other one about 500 m to the west, at the foot of Gellért Hill where the water of Danube had already no significant gravitational effect (see Figure 10). It is important to remark, that the gravity stations are influenced not only by the level of the Danube, but by ground-water fluctuations as well.

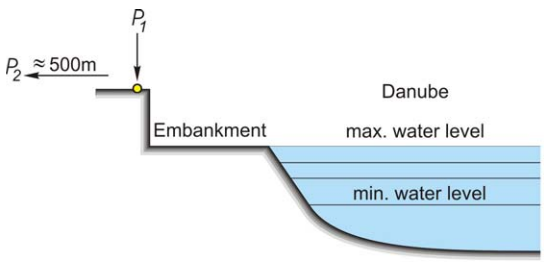

Fig. 10 Sketch for the measurements of Danube River flood

Measurements were carried out by two LCR gravimeters on four subsequent days. Daily observations were carried out in a P_1–P_2–P_1–P_2–P_1–P_2 sequence. The maximum subsidence of the level of the Danube was 4.16 m and the respective gravity change 41 μGal (Csapó, Szabó, Völgyesi 2003).

To verify the measurements, model calculations were carried out applying the software Mod3D written by I. Cerovský (Cerovský et al, 2004). We have calculated the gravitational effect of the water mass of Danube flood using a three-dimensional model. The 3D model of the Danube flood can be seen in Figure 11. The heights of topography and the position of water mass were given a 10×10 m grid spacing. Dimension of the model is 640×640 m. Anomalous gravitational field components and the gravitational tensor elements were computed along the central cross sectional part (*M section*) of this model.

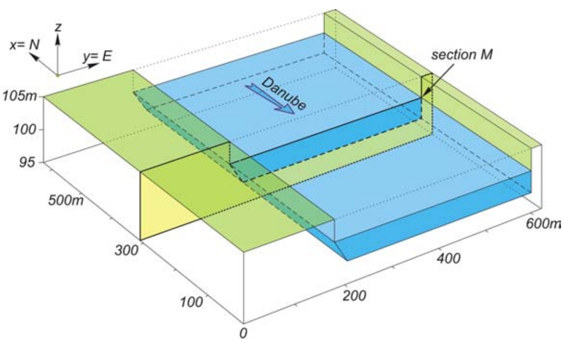

Fig. 11 The 3D model of Danube River's water mass

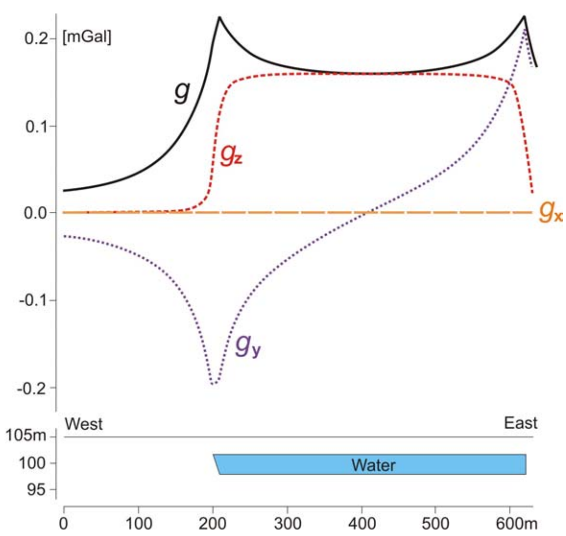

Fig. 12 The gravitational field components $\mathbf{g}(g_x, g_y, g_z)$, computed along the central cross sectional part (*M section*) of the model.

In Figure 12, the gravity effects of the water masses versus distance are plotted in a section perpendicular to the river bank. In the lower part of the figure the section of water mass model is presented, while in the upper part the respective gravity effects. As it can be seen, in the immediate neighbourhood of the river bank the gravity effect can reach a value as high as 150 μGal, and decreases quickly moving away from the river bank. In case of 4 m high flood only a few μGal variation can be registered at a distance of 40-50 m. In point P_1, the gravity effect of 4 m high water mass was found to be about 40 μGal, which is in good agreement with the measured 41 μGal.

Elements of full gravitational tensor were computed too for the water mass of Danube flood. The variations of Eötvös tensor elements W_{xx}, W_{yy}, W_{xy}, W_{zx}, W_{zy}, W_{zz} were computed along the central cross sectional part of the model. Results can be seen in Figure 13. The changes of gradients are very similar to the results referring to the water reservoir; the maximum values can be seen exactly along the river banks.

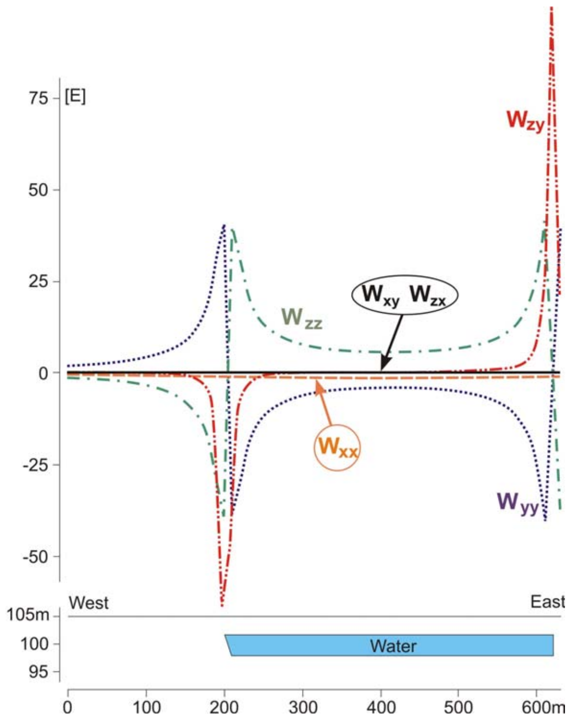

Fig. 13 The full gravitational tensor elements W_{xx}, W_{yy}, W_{zz}, W_{xy}, W_{zx}, W_{zy} in a section perpendicular to the river bank.

3 Conclusion

Investigation of time variation of gravity is important in gravimetry. Two case studies were considered where gravity field changes were detected resulting from water mass variations. One case is an urban water reservoir where the maximum daily change of water is 40000 m^3, the other is the water level fluctuations of the Danube River in Budapest during the great flood in 2002. In the case of water reservoir because of the rapid water level change, the gravity change can reach a value as high as 0.1-0.2 mGal, and gravity gradient changes may be 50-100 E in a few hours. This is the same case for the Danube River, the water level change may cause the same magnitude of variations of gravity and gravity gradients in a few days.

The other important conclusion of our results is: gravity stations should not be located at places where the movement of large volumes of water may be presumed. The measurements and model calculations prove that in the case of gravity base networks, locating stations near river banks or any other places where the changes of water mass may occur, one has to consider the gravity effect of water level fluctuations.

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