NEW MEASUREMENTS FOR THE DETERMINATION OF LOCAL VERTICAL GRADIENTS

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ABSTRACT

Measured vertical gravity gradients usually quite differ from the normal value of 0.3086 *mGal/m* (1mGal = 10^{-5}m/s^2). Generally the changing of vertical gradient is rather big, up to a few 10 *cm* height above the ground and can be taken into consideration as a second order function. Over 40 *m* height the changing is linear and over 500 *m* height the changing is very small, the measured value of vertical gradient is about 0.3073 *mGal/m*.

At present the datum level (*mGal* level) of modern gravity networks are mostly determined by absolute gravity values measured by absolute gravimeters, which values are referred to the reference heights of absolute gravimeters. So the datum level (*mGal* level) of the network of absolute points differs from the datum level of the network of relative measurements. The vertical gradients can be used for the conversion of measured gravity from the reference height of an instrument to a bench mark. So the vertical gradients are playing a key role for joining the two types of network. The height correction is necessary to determine by an accuracy of several *μGal*, not to decrease the reliability of the transformed value of gravity. So the vertical gradients should be determined as high accuracy as it possible and using the normal value of vertical gradient (0.3086 *mGal/m*) is not sufficient for this purpose.

Keywords: **vertical gravity gradients, height correction, transformation between absolute and relative gravity networks**

1. DETERMINATION OF VERTICAL GRADIENTS

Importance of exact local value of vertical gradients (*VG*) was announced in earlier publications (CSAPÓ 1997; CSAPÓ, PAPP 2000) both from the point of view of precise gravity measurements and from the checking of values computed by model computations. Both mass inhomogenities of surrounding environment and the type of approximation method of function *dg/dh* have significant effects for the measured vertical gradient values. It is pointed out, that measuring the vertical gradient only at two points along the perpendicular line of a given point does not provide enough accuracy on the one hand, and the measured vertical gradient values significantly differ from the normal value of 0.3086 *mGal/m* on the other hand (CSAPÓ, VÖLGYESI 2002).

Generally the changing of vertical gradient is rather big, up to a few **10 *cm*** height above the ground and can be taken into consideration as a second order function, but over **40 *m*** height the changing of *VG* can be taken into account as a linear function (RÖDER, WENZEL 1986). According to the aerial measurements in the height interval of **500-1700 *m*** the changing of vertical gradient is very small (it is only 1-2%), the measured value is about **0.3073 *mGal/m*** (HAMMER, GUMERT 1984).

Based on our earlier test measurements and experiences a method was processed for field measurements of vertical gradients. These measurements give a suitable accuracy for any practical purposes.

Two LCR-G gravimeters and a special tripod were applied for these measurements (CSAPÓ, VÖLGYESI 2002). Heights of measurements have been setting as the sensing mass of instruments were at special heights above the benchmark (A = 50 mm, B = 700 mm and C = 1300 mm) (BECKER et al 2002). Nine series were measured by each gravimeter with a sequence of ***A-B-C-B-A-B-C-B-A***. All LCR gravimeters were calibrated at a calibration line. Periodic errors of gravimeters by 1 mGal at the processing of measurements were taken into account, and gravimeters were equipped by electronic levels. The readings were made by spur wheel and digital voltmeter connected to the output of Capacitance Beam Position Indicator (***CPI***). Interpolation method was used for three readings of spur wheel and digital voltmeter. Duration of measuring sequence for each gravimeter were about 60 minutes, measurements were made by two instruments, one after the other. The observations were repeated on three different days at each point therefore six independent measured sequences could be processed for every point. In this way the vertical gradients can be computed from the corrected relative values of gravity measured by LCR gravimeters at different heights above a benchmark. The applied corrections can be found in the references.

Measurements were made in the years of 2002-2003 at different sites of Budapest. Coordinates of points, height reductions supposing linear *VG*(1) and quadratic *VG*(2) vertical gradient can be found in *Table 1*. If the variation of gravity is a linear function of height is supposed to be linear, the height reduction *VG*(1) can be computed by

$$VG(1) = \underbrace{\partial g / \partial h}_{VG} \cdot \Delta h \; .$$

If the variation of gravity is a quadratic function of height the height reduction *VG*(2) can be computed by

$$VG(2) = \partial g / \partial h \cdot \Delta h + \partial^2 g / \partial h^2 \cdot \Delta h^2$$

In *Table 1* the *VG*(1) and *VG*(2) are the height reductions referring to the height of 1*m* above the benchmark.

In spite of the few measurements can be stated that there are usually no significant differences between the linear and quadratic height reductions at some points, but there can be significant differences at other ones.

Especially striking poor reliability can be seen at point *VT (Gellért-hegy)* which point was located just above an urban water reservoir. The poor reliability can be explained by the significant daily water-level fluctuation in the water reservoir - the daily changing of water mass may be a few 10000 m³ (CSAPÓ, SZABÓ, VÖLGYESI 2003). Therefore this continuously changing water mass causes the bigger dispersion of vertical gra-

dient values at *VT* point measured at different days and different parts of the days, than at the other points.

points	name of ponits	φ	λ	H[m]	$\delta g_h(1)$	$\delta g_h(2)$
2143	Hármashatárhegy	47-33-23	19-00-10	463	−386.0 ± 10	−393.5 ± 5
2142	Táborhegy	47-33-01	19-00-31	412	−350.0 ± 14	−354.7 ± 7
2141	Remetehegy	47-32-39	19-00-39	354	−350.6 ± 6	−351.3 ± 5
2140	Szépvölgyi út	47-32-27	19-00-29	283	−284.9 ± 6	−288.2 ± 6
821	Mátyáshegy	47-32-00	19-00-57	201	−258.7 ± 8	−259.0 ± 8
2139	Rózsadomb	47-31-02	19-01-53	153	−304.2 ± 8	−306.0 ± 4
17	BME kert	47-28-50	19-03-35	105	−300.1 ± 6	−305.3 ± 4
VT	Gellérthegy	47-29-24	19-02-32	160	−326.9 ± 21	−332.8 ± 12
MET	Pestlőrinc	47-25-48	19-11-01	137	−303.3 ± 9	−303.1 ± 6
OMH	Németvölgyi út	47-30-00	19-00-01	142	−293.5 ± 6	−293.3 ± 5

It's worth mentioning that previous vertical gradient measurements were made previously on two different heights at point 821 (Mátyáshegy) have given 262.5 ± 4 μGal for the value of VG (1).

An example can be seen in *Table 2* for the results of different vertical gradient measurements on three different heights at point 2139 *(Rózsadomb).*

measurements	LCR-1919		LCR-963	
	$\delta g_h(1)$	$\delta g_h(2)$	$\delta g_h(1)$	$\delta g_h(2)$
1.	− 309.4	− 312.7	− 300.0	− 307.5
2.	− 301.7	− 307.6	−306.7	− 301.0
3.	− 303.2	− 307.4	− 303.8	− 299.3
mean (1.-3.)	− 304.8 ± 6	− 309.2 ± 2	− 303.5 ± 8	− 302.7 ± 4
average (1919 & 963)	−304.2 ± 8	− 306.0 ± 4		

It can be easily seen, that there is no significant difference between the height reductions determined by linear or quadratic approximation, but the reliability of *VG*(2) is better than of *VG*(1) for both gravimeters and for the all measurements. Moreover it can be seen from *Table 2* that the difference of height reductions determined from the measurements of different gravimeters can be as high as 7*μGal,* independently of the approximation method. So this justifies that a single gravimeter does not give significant reliability of local vertical gradient's values. The reason of the various discrepancies of

computed *VG* values from different measurement's series is the changing effect of the varying wind, air pressure, temperature, microseismic effects, etc. These enumerated effects may cause various drift of gravimeters therefore the measurements made by certain gravimeters on the same days may be more or less different. Similar problem may come from the transporting vibration of gravimeters, or the discrepancies of measurements may come from the reason of not taking into account the periodic errors of gravimeters by 1 *mGal*.

Results of two different adjustment versions of vertical calibration basis of Budapest can be seen in *Table 3* measured in 2003. Fixed values of adjustment were the gravity of absolute station of Budapest (82) and the station of Budaörs airport (107.10). Vertical gradient values of these points had been determined earlier, before the adjustment of Hungary's New Gravity Base Network (MGH-2000). The height reduction of gravimeters have been taken into consideration using the normal value of vertical gradient (0.3086 *mGal/m*) in the case of adjustment version "*A*", but the measured value from observation on three different heights (supposing the vertical gradient *dg/dH* as a linear function of height) in the case of adjustment version "*B*".

Table 3. **Different adjustment versions of vertical calibration basis of Budapest**

point	A	VG	B	VG	$\delta\Delta g_{(B-A)}$
	mGal	mGal/m	mGal	mGal/m	mGal
2139	835.3946		835.3962	− 0.3042	0.0016
2140	809.9906		809.9921	− 0.2849	0.0015
2141	796.0111		796.0152	− 0.3506	0.0041
2142	783.8461	− 0.3086	783.8501	− 0.3500	0.0040
2143	771.1283		771.1347	− 0.3860	0.0064
82				− 0.2519	
107.10				− 0.3084	

It can be seen from *Table 3*, that demonstrable differences of adjusted values of points - measured by high precision gravimeters and computed by the two types of height reduction values - are possible. There was only maximum 1 *μGal* difference between the height reduction values computed by linear or quadratic approximation because of the small heights (50-120mm) of instruments.

2. ESTABLISHING THE CONNECTION BETWEEN ABSOLUTE AND RELATIVE GRAVITY NETWORKS BY VERTICAL GRADIENTS

The datum level (*mGal*-level) of modern gravity base networks is defined by gravity values determined by absolute measurements. These absolute points are the zero order network points in most of European countries, and form the framework of UEGN-2000 network too. Gravity values measured by absolute and relative gravimeters are referring to the reference height of the instrument above the bench mark. Each gravimeter has special reference height therefore the different networks of absolute and relative points have different datum level.

There are two possible methods of conversion between the two types of gravity networks.

In the case of first method the vertical gradient values referring to the absolute points must to be determined with some kind of procedure. The measured vertical gradient values depend partly on the method of determination partly on the approximation method of *VG* function (linear or quadratic) as the examples show in literature and in this paper. Differences of height reductions depending on the applied approximation method may often be larger than the accuracy of measurements by absolute gravimeters. At the same time application of *VG*'s normal value for reducing the measured gravity to the bench mark is not suitable, because according to the effect of mass inhomogenities next to the bench mark quite different *VG* value have to be applied. In this case the technical parameters of *VG*'s determination (linear or quadratic approximation method, instrument heights) have to be uniformly applied for all gravity points.

The other method of conversion between the two types of gravity networks is presented by the help of Fig. 1.
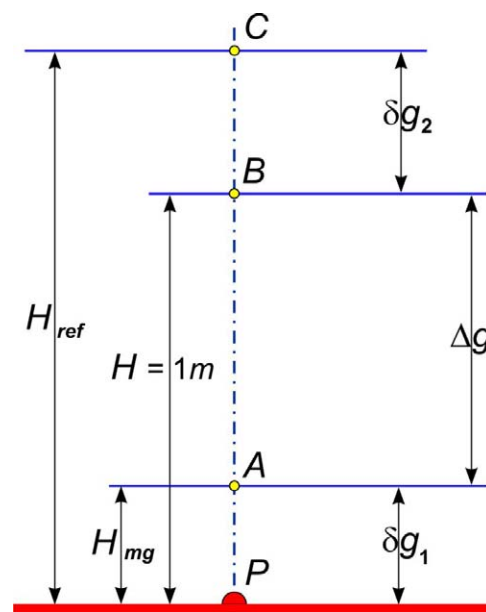

Fig. 1. Definition of different heights for the conversion of *g*

First of all a height should be chosen on the perpendicular line of point P which is the average reference height of absolute gravimeters. The reference height of absolute gravimeters can be found between about 80 and 110 cm depending on the construction of instruments, therefore H=100 cm can be chosen for the average height (the distance between the point P and B is 100 cm). In practice the reference height of an absolute gravimeter is at the height of point C. At the same time the reference height of a relative gravimeter is at the height of $H_{mg} = 5 - 10\,cm$ therefore the sensing mass of the instrument is at the height of point *A* (there is no possibility to set up the gravimeters so that the level of sensing mass would be at the level of point *P*). Determining the vertical gradient by measurements, the sensing mass of the relative gravimeter is at this height of point *A* as well, at the lowest position of the instrument. Consequently the *g* referring to point *C* measured by absolute gravimeter can be convert to point *P* so that the gravity difference Δg should be determined between the point *A* and *B*, then δg_1, δg_2 height correction should be added to it:

$$g_P = g_C + \Delta g + \delta g_1 \pm \delta g_2 \ ,$$

where δg_1 is the height correction between the level of the sensing mass of a relative gravimeter and the bench mark (between point A and P), and δg_2 is the height correction between the real and the average reference height of absolute gravimeters (between the point B and C). As the vertical gradient value has been determined by some kind of method, then height corrections of δg_1 and δg_2 can be computed as well. There is no significant difference larger than the demand of accuracy between the height reduction δg_1 computed by linear or quadratic approximation because the distance PA is only 5-15 *cm*. The situation is the same with height reduction δg_2 because the height difference between the real and the average reference height of absolute gravimeters (between the point B and C) is only 10-20 *cm*. So the maximum difference is 1 *μGal* between the height reduction δg_2 computed by linear or quadratic approximation for the distance *BC*. Based on the decision of the specialists working on the adjustment of Unified European Gravity Net (UEGN-2000) we have applied this latter type of conversion.

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