

DATA PROCESSING OF QDAEDALUS MEASUREMENTS

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Abstract. QDaedalus is an automated, computer-controlled astrogeodetic measurement system which has been developed especially for the determination of the local vertical deflections. It is possible with the system in 20-30 minutes of measurement time at night to get direction of the local Zenith accurate to 0.1 seconds of arc based on the joint inversion of measurements of the total station and position of stars on the CCD. Two iterative robust inversion procedures has been tested: (1) the so-called least-squares Danish method and (2) most frequent value procedure with Cauchy-Steiner weights. Our evaluation has shown that method (2) achieves better performance based on more than 70 series of measurements at the same station.

1. INTRODUCTION

In geodesy and in civil engineering it is important to know the gravity field. For example, physical heights related to equipotential surfaces of gravity are required for certain engineering projects. Intrinsically linked to the gravity field are the local horizon and its normal vector, the local zenith direction (direction of the gravity vector or of the plumb line). Zenith direction is usually referenced to an ellipsoidal normal vector of a geodetic reference system. Deviations of the zenith direction from the ellipsoidal normal vector in North-South and East-West directions are the components (ξ, η) of the deflection of the vertical (DOV). These components can be determined by astrogeodetic methods.

High accuracy astrogeodetic DOV determination was a very time-consuming and expensive field procedure in the past. By high accuracy we mean 0.1" precision of DOV components. Recent advances in instrumentation and computer technology speeded up DOV determination considerably by digital zenith cameras. These are, however, quite expensive, dedicated field instruments with high resolution tiltmeters and very sensitive CCD sensors [1]. On the other hand, it is relatively straightforward to modify a modern robotic total station to be suitable for astrogeodetic DOV determination. QDaedalus is such a system [2].

Accuracy of DOV determination depends on several factors. Assuming a standard data processing work-flow, HIRT [3] showed that in case of digital zenith cameras the main accuracy factor is the number of independent star observations. That is by increasing the number of observations the accuracy can be increased, too. No such analysis exists for astrogeodetic DOV determinations by robotic total stations, however. A second open question is whether standard least-squares data processing is optimal for achieving the desired accuracy with minimum number of data (or measurement time). As it is known, least-squares procedures are optimal only for Gaussian data distributions. Therefore, optimality of data processing depends critically on the distribution of measurements.

Our present work addresses these issues. We analyzed the distribution and accuracy of QDaedalus measurements and compared two different data processing methodologies in terms of their accuracy. The first one was a standard robust least-squares inversion for the unknown parameters (the Danish method). The second one was a new procedure of robust inversion using Cauchy-Steiner weights.

In Section 2 and 3 an overview of the QDaedalus system and the measurement process is given. Section 4 discusses the inversion problem of DOV determination and its solution with the two methods mentioned above. In the next Section we present numerical tests with the two methods and in Section 6 some conclusions are drawn.

2. THE QDAEDALUS SYSTEM

The QDaedalus system is a computer-controlled, GNSS-assisted automatic measuring system, which can be used mainly for high-precision determination of the local vertical direction (deflection of the vertical) [2]. The main system component is an adapted Leica TCA 1800 robotic total station. The eyepiece of the instrument is replaced with a CCD sensor, hence instead of visual observation the image of the telescope can be viewed on the computer monitor. Focussing is also computer-controlled by using an attached servo motor [4].

The GNSS receiver serves two purposes: first, it provides accurate timing of star observations for both the CCD sensor and control computer; second, it measures and sends WGS84 coordinates to the computer through an RS232-USB adapter. Large volumes of CCD image data are transmitted through a high-speed (hot) line into the computer via a PCMCIA or EXPRESS CARD interface. The external computer controls the robotic total station through another dedicated channel via another RS232-USB adapter and the interface unit of the total station (Figure 1).

The QDaedalus software controls the whole system and processes data. In particular it guides the robotic total station, telescope focus, receives and processes CCD images, GNSS data; in case of astrogeodetic measurements computes topocentric star coordinates, maintains a measurement database and in situ calculates deflection of the vertical components.

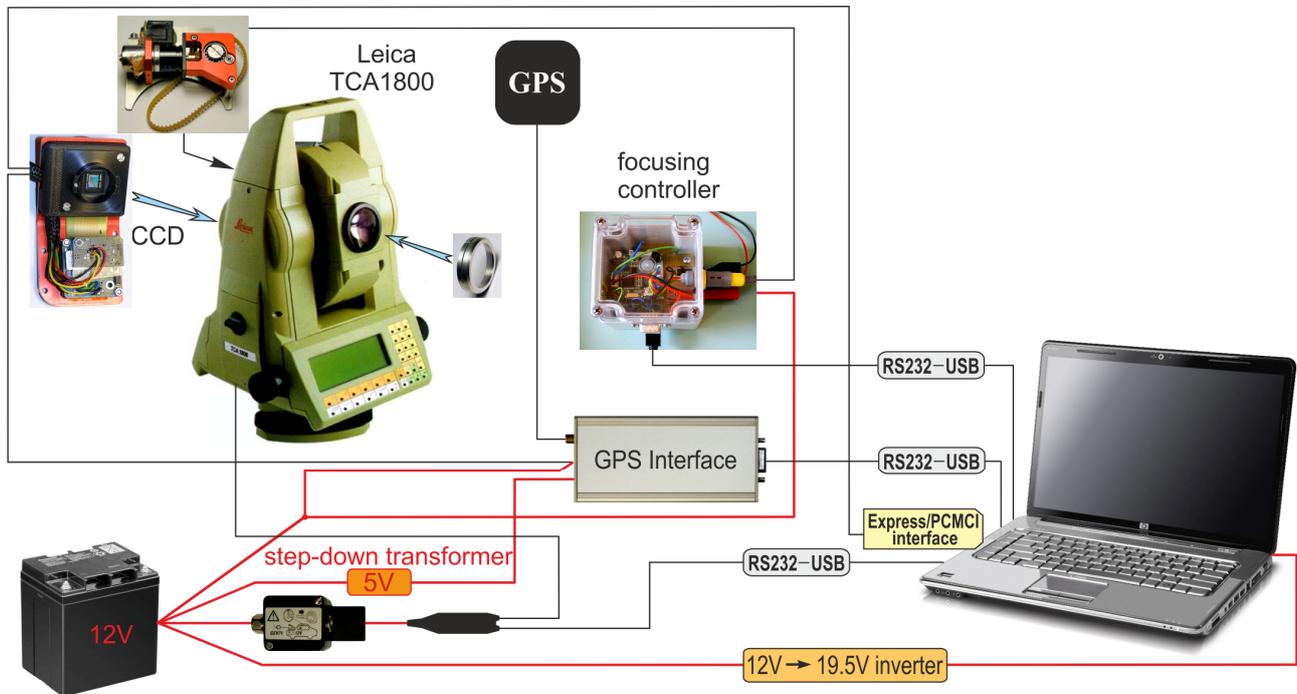


Figure 1
Schematic construction of the QDaedalus system

3 MEASUREMENT OF VERTICAL DEFLECTIONS BY QDAEDALUS SYSTEM

Deflection of the vertical (its components) can be computed from geodetic coordinates of the local vertical and ellipsoidal normal [4]:

$$\xi = \Phi - \varphi, \quad (1)$$

$$\eta = (\Lambda - \lambda) \cos \varphi. \quad (2)$$

GNSS measurements provide geodetic latitude and longitude φ, λ with respect to the WGS84 ellipsoid, whereas astrogeodetic observations provide astronomical latitude and longitude Φ, Λ

based on known celestial equatorial coordinates (δ, α) of measured stars [5].

The starting point of astrogeodetic methods is the known celestial equatorial coordinates of observed stars in the quasi-inertial barycentric ICRS system (International Celestial Reference System). Our goal is to determine our position (or local vertical direction) Φ, Λ in the ITRS system (International Terrestrial Reference System), hence we need to transform coordinates of stars δ, α from ICRS into ITRS.

This in practice requires two steps: first we determine local zenith direction with respect to the measured stars; this direction is characterized by the coordinates $\alpha_{Zenit}, \delta_{Zenit}$. In the second step zenith direction in the celestial equatorial system is transformed into the ITRS system by a coordinate transformation. The relationship between celestial and terrestrial coordinates is shown in Figure 2.

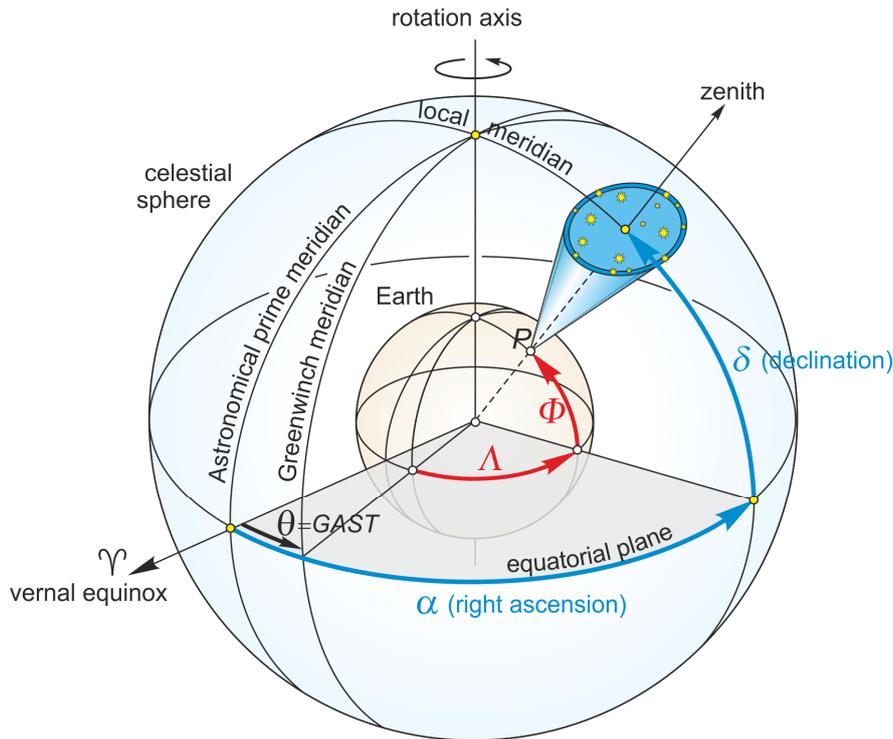


Figure 2
Principle of DOV measurement

Stars very close to zenith are measured with traditional zenith cameras, whereas QDaedalus measures stars along a celestial small circle centered on zenith. This circle can be visualized as the intersection of the celestial sphere with a circular cone of half apex angle e.g. 30° with its axis pointing at zenith. In either case, direction of the local vertical (zenith) is determined from the star measurements.

Due to the software control mentioned, during measurement the pointing axis of the telescope is set to the positions of stars of the FK6 star catalogue computed for the particular observation station and epoch. Stars are selected to have a possibly uniform distribution along the mentioned celestial small circle. Night measurement with QDaedalus is show on Figure 3.

4. INVERSION PROCEDURE

Standing axis of a total station ideally points to the local Zenith direction (Figure 4). Readings (ℓ, z) to the pointing axis of the telescope are given in the local horizontal-vertical (Hz-V) system. Position (x, y) of the star's image on the CCD sensor can also be determined automatically by image processing. Thereby it is possible to reconstruct (with known calibration constants) the direction to any measured star in the local horizontal-vertical system.



Figure 3
Night measurement by the Qdaedalus system

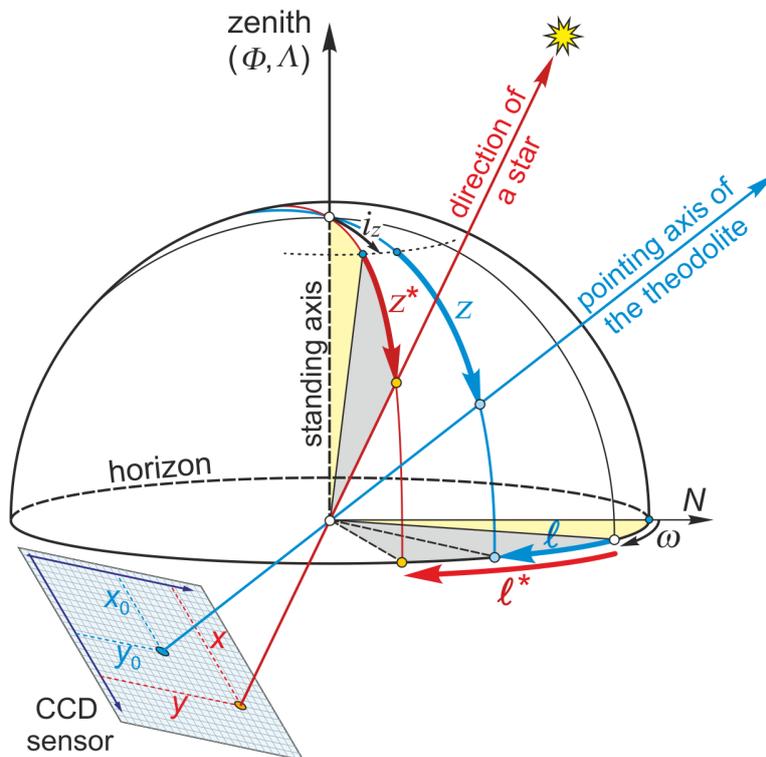


Figure 4
Model of local zenith direction determination

In Section 3 it was mentioned that the position of stars for any measurement epoch can be calculated from barycentric ICRS coordinates in the Earth-fixed ITRS system. By transforming these ITRS coordinates to the local Hz-V system and taking into account astronomical refraction we can compare measured and computed (ℓ^*, z^*) Hz and V directions. These relations are expressed by the following two model equations Eq. (3).

$$\left. \begin{aligned} (\ell - \ell^*) \sin z^* - a_{11}(x - x_0) - a_{12}(y - y_0) &= 0 \\ z - z^* - a_{21}(x - x_0) - a_{22}(y - y_0) &= 0 \end{aligned} \right\} \quad (3)$$

where $x_0, y_0, a_{11}, a_{12}, a_{21}, a_{22}$ are calibration constants and

$$\left. \begin{aligned} \ell^* &= f_\ell^*(\Phi, \Lambda) - \omega \\ z^* &= f_z^*(\Phi, \Lambda) - i_z - \delta_{refr} \end{aligned} \right\} \quad (4)$$

are functions of the model parameters. Our goal is to determine the model parameter vector

$$\vec{m} = \{\Phi, \Lambda, \omega, i_z\}$$

where $\Phi, \Lambda, \omega, i_z, \delta_{refr}$ denote astronomical latitude, longitude, orientation correction, index error of the instrument and refraction correction, respectively.

We have measurements for each epoch i of the telescope axis and position of the star's image on the CCD sensor:

$$\vec{d}_{mi} = \{\ell_i, z_i, x_i, y_i\}, \quad i = 1, 2, \dots, N,$$

where N denotes the number of measurement epochs.

Model equations (3) can be written for i -th measurement epoch as

$$\left. \begin{aligned} [\ell_i - \ell_i^*(\Phi, \Lambda, \omega)] \sin z_i^*(\Phi, \Lambda, i_z) - a_{11}(x_i - x_0) - a_{12}(y_i - y_0) &= 0 \\ z_i - z_i^*(\Phi, \Lambda, i_z) - a_{21}(x_i - x_0) - a_{22}(y_i - y_0) &= 0 \end{aligned} \right\}. \quad (5)$$

General form of the above equations is $\vec{g}_i(\vec{d}_{ei}, \vec{m}) = 0$, where $\vec{d}_{ei} = \{\ell_i, z_i, x_i, y_i\}$ denote theoretical values. The model equations are implicit in both data and parameters. Theoretical values cannot be determined uniquely from model equations since there are $2N$ model equations for $4N + 4$ unknowns, therefore our problem is underdetermined. We can introduce $4N$ measurement residuals $\vec{e}_i = \vec{d}_{ei} - \vec{d}_{mi}$ instead of theoretical values, and we still have an underdetermined problem. The usual solution to this problem, discussed by e.g. Vanicek and Krakiwsky [6], is to minimize either weighted residuals (length of \vec{e} in data space)

$$E = \vec{e}^T \underline{\underline{W}} \vec{e} = \min \quad (6)$$

or its weighted projection $\vec{e}' = \underline{\underline{B}} \vec{e}$,

$$E' = \vec{e}'^T \underline{\underline{W'}} \vec{e}' = \min \quad (7)$$

in model space under constraint $\bar{g}(\bar{d}_e, \bar{m}) = 0$. We have a constrained minimization problem, which can be solved by the method of Lagrange multipliers. We introduce a vector $\bar{\lambda}$ of Lagrange multipliers or Lagrange correlates from the model space and add the scalar product of Lagrange multipliers and vector of constraints to the minimum condition

$$\Phi(\bar{e}, \bar{m}, \bar{\lambda}) = \bar{e}^T \underline{\underline{W}} \bar{e} + \bar{\lambda}^T \bar{g}(\bar{d}_m + \bar{e}, \bar{m}) = \min. \quad (8)$$

It has been shown by HADLEY [7] that the addition of the scalar product does not change the location of the extremum. Using gradient method (linearization) to solve for the extremum yields

$$\left. \begin{aligned} \frac{\partial \Phi}{\partial \bar{e}} &= 2\bar{e}^T \underline{\underline{W}} + \bar{\lambda}^T \frac{\partial \bar{g}}{\partial \bar{d}_e} = 2\bar{e}^T \underline{\underline{W}} + \bar{\lambda}^T \underline{\underline{B}} = 0 \\ \frac{\partial \Phi}{\partial \bar{m}} &= \bar{\lambda}^T \frac{\partial \bar{g}}{\partial \bar{m}} = \bar{\lambda}^T \underline{\underline{G}} = 0 \\ \frac{\partial \Phi}{\partial \bar{\lambda}} &= \bar{g}(\bar{d}_m + \bar{e}, \bar{m}) \approx \underline{\underline{B}} \bar{e} + \underline{\underline{G}} \delta \bar{m} + \bar{g}_0 = 0 \end{aligned} \right\}. \quad (9)$$

If we introduce Jacobian matrices of partial derivatives of function: \bar{g}

$$\left. \begin{aligned} \frac{\partial \bar{g}}{\partial \bar{d}_e} &= \underline{\underline{B}} \\ \frac{\partial \bar{g}}{\partial \bar{m}} &= \underline{\underline{G}} \end{aligned} \right\}, \quad (10)$$

the following linear system of equations must be solved for determination of minimum

$$\left. \begin{aligned} \underline{\underline{W}}^T \bar{e} + \underline{\underline{B}}^T \bar{\lambda} &= 0 \\ \underline{\underline{G}}^T \bar{\lambda} &= 0 \\ \underline{\underline{B}} \bar{e} + \underline{\underline{G}} \bar{m} + \bar{g}_0 &= 0 \end{aligned} \right\}, \quad (11)$$

where we substituted \bar{m} for the change of the parameter vector $\delta \bar{m}$.

The inversion solution for the change of the parameter vector is

$$\bar{m} = \left(\underline{\underline{G}}^T \left(\underline{\underline{B}} \underline{\underline{W}}^{-1} \underline{\underline{B}}^T \right)^{-1} \underline{\underline{G}} \right)^{-1} \underline{\underline{G}}^T \left(\underline{\underline{B}} \underline{\underline{W}}^{-1} \underline{\underline{B}}^T \right)^{-1} \bar{g}_0 = \underline{\underline{G}}^{-gh} \bar{g}_0, \quad (12)$$

where we introduced on the right hand side the generalized inverse matrix $\underline{\underline{G}}^{-gh}$.

The result of inversion obviously depends on the chosen weight matrix $\underline{\underline{W}}$. In the next two subsections we discuss two different iterative weighting schemes which aim at robust and resistant solution of the inversion. But first it seems appropriate to include a brief review on notions of robustness and resistivity following STEINER [8].

Robustness refers to the high efficiency of estimation for a whole family of distributions (supermodels). Resistivity means that our estimation is resistant to outliers. STEINER [8] discussed statistical efficiency of several estimation methods. He obtained for the $f_a(x)$ supermodel the

efficiencies visualized in Figure 5. We can see that method (1), P_k norm (Cauchy-Steiner weights) has higher statistical efficiency for this supermodel than method (2), the so-called Danish method [9] with parameter $c = 3$. We can see that indeed the Danish method has 100% efficiency for Gaussian distribution, but its efficiency decreases sharply and has approximately only 50% for long-tailed Cauchy distribution. In Section 5 we will test QDaedalus measurement residuals to see if they follow a Gaussian distribution.

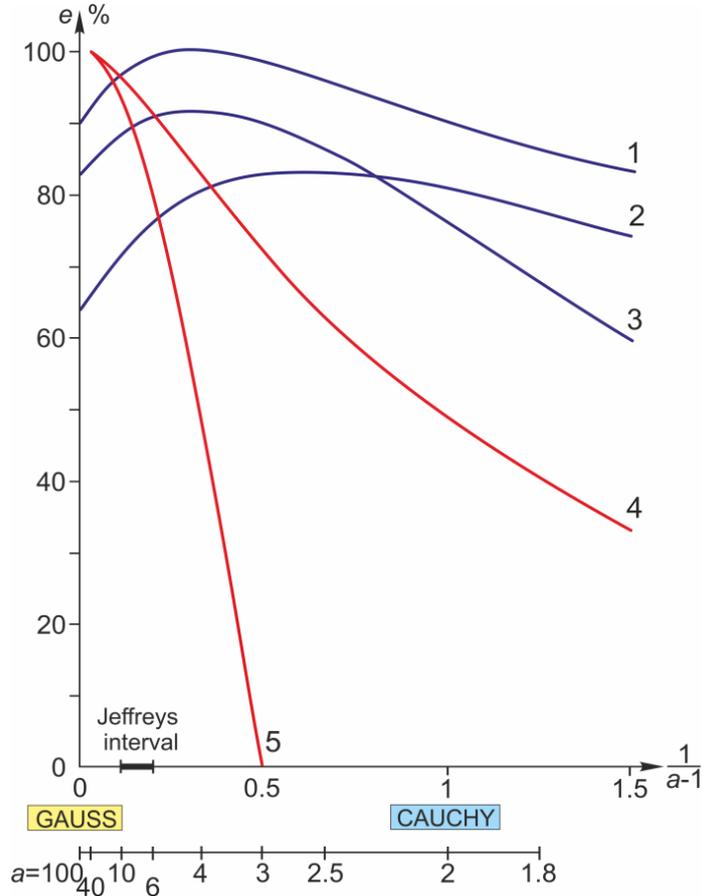


Figure 5

Statistical efficiencies (e %) of some estimation procedures for a large range of probability distribution types (the supermodel $f_a(x)$). Same as Fig. 2 of STEINER [8]. 1: P_k norm, $k=1.9$; 2: median; 3: Danish method, $c=1.5$; 4: Danish method, $c=3$; 5: L_2 norm

4.1 Robust inversion by the Danish method

Resistance to outliers (robustness) can be ensured by iteratively re-computing weights of residuals according to the so-called Danish method by KRARUP and KUBIK [9]. Weights of the Danish method are defined by the following formulas

$$w_{k+1} = w_k f(e_k)$$

$$f(e_k) = \begin{cases} 1, \text{ if } \frac{|e_k| \sqrt{w_0}}{\sigma_0} < c \\ \text{else } \exp\left(-\frac{|e_k| \sqrt{w_0}}{c \sigma_0}\right) \end{cases}, \quad (13)$$

where k is the iteration number, e_k is residual after step k , σ_0 is square root of the variance factor, w_0 is initial weight and c is constant (usually equals to 3).

4.2 Robust inversion by Cauchy-Steiner weights

Our aim is minimization of residuals \vec{e}' according to Eq. (7) by using Cauchy-Steiner weights. Projection $\vec{e}' = \underline{\underline{B}} \vec{e}$ transfers residuals from measurement space into model space. Constrained minimization of the logarithm of the P_k -norm [10]

$$P_k(\vec{x}) = \varepsilon \left\{ \prod_{i=1}^n \left[1 + \left(\frac{x_i}{k\varepsilon} \right)^2 \right] \right\}^{\frac{1}{2n}} \quad (14)$$

where ε is the dihesion of the deviations x_i defined by

$$\varepsilon^2 = \frac{3 \sum_{i=1}^n \frac{x_i^2}{(\varepsilon^2 + x_i^2)^2}}{\sum_{i=1}^n \frac{1}{(\varepsilon^2 + x_i^2)^2}} \quad (15)$$

leads by using the method of Lagrange multipliers to the equation

$$\Phi(\vec{e}, \vec{m}, \vec{\lambda}) = \ln P_k(\vec{e}') + \vec{\lambda}^T \vec{g}(\vec{d}_m + \vec{e}, \vec{m}) = \min. \quad (16)$$

Finding minimum requires vanishing derivatives again by using the gradient method and leads to the equations

$$\left. \begin{aligned} \frac{\partial \Phi}{\partial \vec{e}} &= \frac{\partial \ln P_k(\vec{e}')}{\partial \vec{e}} + \vec{\lambda}^T \underline{\underline{B}} = \frac{1}{n} \vec{e}^T \underline{\underline{B}}^T \underline{\underline{W}}' \underline{\underline{B}} + \vec{\lambda}^T \underline{\underline{B}} = 0 \\ \frac{\partial \Phi}{\partial \vec{m}} &= \vec{\lambda}^T \underline{\underline{G}} = 0 \\ \frac{\partial \Phi}{\partial \vec{\lambda}} &= \underline{\underline{B}} \vec{e} + \underline{\underline{G}} \vec{m} + \vec{g}_0 = 0 \end{aligned} \right\} \quad (17)$$

where

$$\underline{\underline{W}}' = \left\langle \dots, \frac{(k\varepsilon')^2}{(k\varepsilon')^2 + e_i'^2}, \dots \right\rangle \quad (18)$$

is diagonal matrix of Cauchy-Steiner weights belonging to residuals \vec{e}' .

The following linear system of equations must be solved to get the optimal parameter vector

$$\left. \begin{aligned} \underline{\underline{B}}^T \underline{\underline{W}}' \underline{\underline{B}} \vec{e} + \underline{\underline{B}}^T \vec{\lambda} &= 0 \\ \underline{\underline{G}}^T \vec{\lambda} &= 0 \\ \underline{\underline{B}} \vec{e} + \underline{\underline{G}} \vec{m} + \vec{g}_0 &= 0 \end{aligned} \right\}. \quad (19)$$

Hence inversion solution will be the same as above if we make the substitution $\underline{\underline{B}}^T \underline{\underline{W}}' \underline{\underline{B}} \rightarrow \underline{\underline{W}}$

$$\bar{\underline{m}} = \left(\underline{\underline{G}}^T \left(\underline{\underline{B}} \underline{\underline{W}}^{-1} \underline{\underline{B}}^T \right)^{-1} \underline{\underline{G}} \right)^{-1} \underline{\underline{G}}^T \left(\underline{\underline{B}} \underline{\underline{W}}^{-1} \underline{\underline{B}}^T \right)^{-1} \bar{\underline{g}}_0 = \underline{\underline{G}}^{-gh} \bar{\underline{g}}_0 . \quad (20)$$

Therefore we have obtained robust solution to our inversion problem with Cauchy-Steiner weights. Our experience showed that this formulation yielded good distribution of residuals. On the contrary, when we applied Cauchy-Steiner weights for original residuals \bar{e} (i.e. not projected) we could not obtain a good distribution of residuals (certain set of residuals always became very small at the expense of others). This may be due to the application of the P_k -norm instead of the usual least-squares norm.

5. INVERSION TEST RESULTS

At station called Pistahegy, located in the southern part of Budapest, we have more than 70 series of DOV measurements taken at 33 different nights. Average measurement time of each observation is about 50 min. We have processed these observations by the two different inversion methods discussed above. The parameter of the P_k -norm was $k = 1$ in all our tests. DOV components in the GRS80 reference system (MORITZ [11]) and their estimated uncertainties (standard deviations) obtained by both methods are shown on Figure 6.

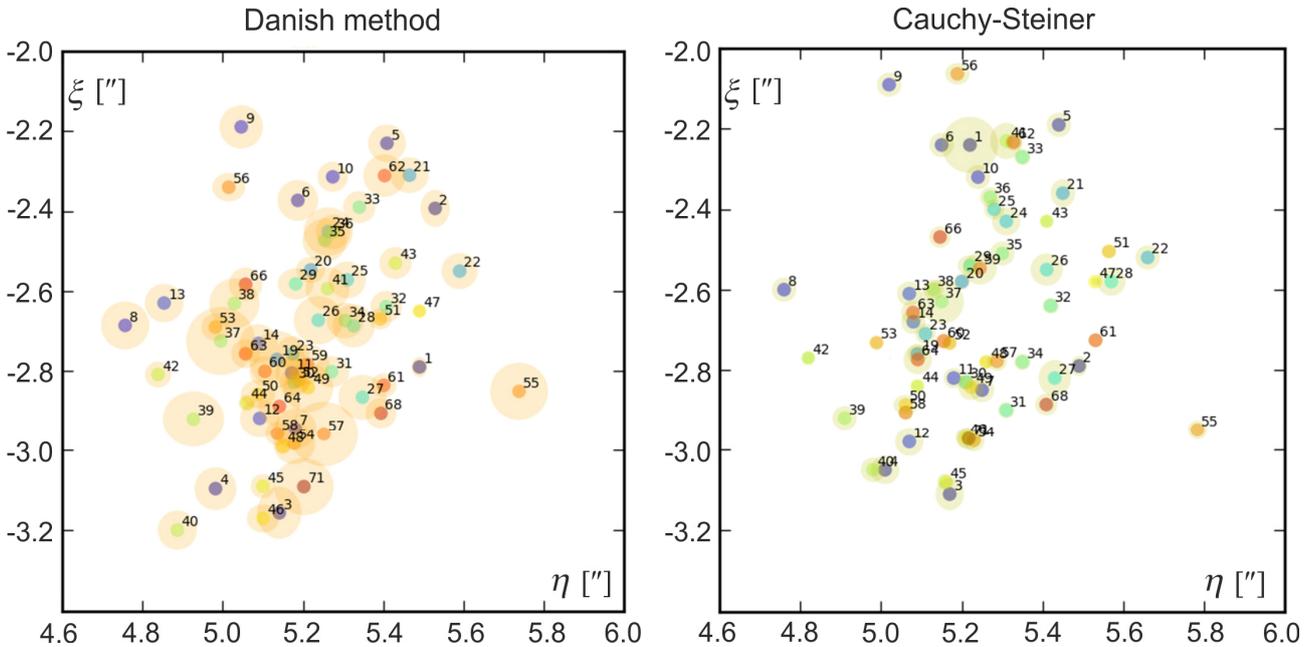


Figure 6

DOV inversion results of 72 QDaedalus measurement series by Danish method vs. Cauchy-Steiner weights at station Pistahegy. Outlying observations are not shown. Numbers of series are given on the plots. Ellipses indicate estimated inversion errors

It is clear from Figure 6 that inversion with Cauchy-Steiner weights generally gives smaller standard deviation of DOV components. Useful information on the quality of inversion is the correlation norm, the average of off-diagonal elements of the parameter correlation matrix. The distribution of the absolute value of the correlation norm for the two inversion methods are shown on Figure 7. We can see that inversion by Cauchy-Steiner weights usually gives smaller correlation between the parameters.

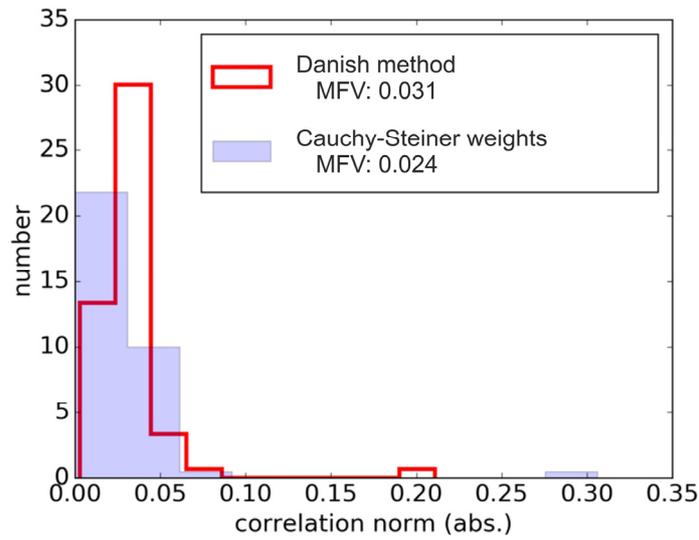


Figure 7

Distribution of absolute value of the correlation norm of DOV inversion results of 72 QDaedalus measurement series by Danish method vs. Cauchy-Steiner weights at station Pistahegy.

5.1 Distribution of inversion residuals

We made tests whether the distribution of inversion residuals of the Danish method follow Gaussian distribution. Figure 5 shows that if this is the case, Danish method gives 100% efficiency of estimation. We have performed one-sample Kolmogorov-Smirnov (K-S) tests to see whether the distribution of inversion residuals follows a Gaussian theoretical distribution. Empirical cumulative distribution (ECDF) of residuals for both vertical (V) and horizontal (Hz) residuals were computed and best fitting Gaussian CDFs are determined. It is important to take into account the fact that when the distribution parameters are estimated from data, the K-S test will be distribution dependent (KEUTELIAN [12]). Hence proper test statistic distributions and confidence levels have to be determined by Monte-Carlo simulation.

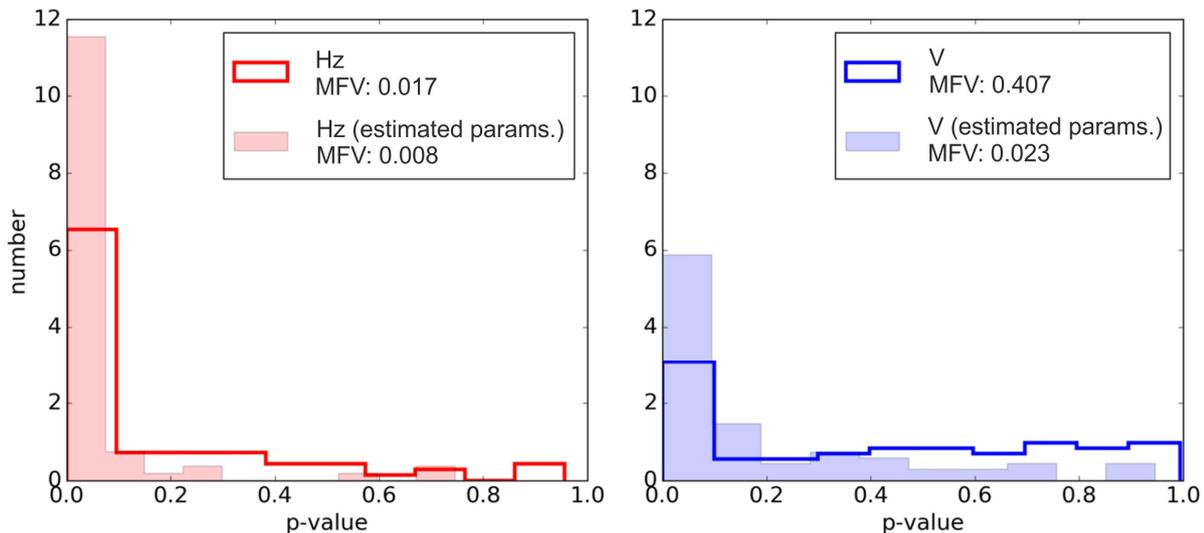


Figure 8

Distribution of p -values of the Kolmogorov-Smirnov test with fixed and estimated parameters for both horizontal and vertical angle measurement residuals for 72 measurement series. Most frequent values (MFV) of p -values are also shown. Monte-Carlo simulation was done using 10,000 samples

The distribution of K-S test statistic p -values are shown in Figure 8 for horizontal and vertical measurement residuals. The most frequent values of the K-S test p -values in case of estimated

parameters are 0.8% and 2.3% for horizontal and vertical measurement residuals, respectively. These low probabilities indicate that inversion results actually do not follow a Gaussian distribution for these 72 QDaedalus measurements. If, instead, we mistakenly treat distribution parameters (mean, std of Gaussian) as fixed, these probabilities will increase to 1.7% and 40.7%, respectively. This may lead to the (false) acceptance of a Gaussian PDF, at least for V residuals.

These test results are in accordance with our observation that Cauchy-Steiner inversion with P_k -norm yields consequently better results than the traditional least-squares estimation applied in geodesy, even if it is made resistant with a re-weighting scheme like the Danish method. M-inversion gives higher statistical efficiencies for distributions significantly differing from Gaussian distribution (cf. Figure 5).

5.2 Accuracy dependence on the number of measurement epochs

If the law of large numbers is fulfilled, it is expected that by increasing the number of observations the accuracy can be increased, too. Hence we made tests to see, that when the number of star measurements (number of measurement epochs) is increased, how this affects the estimated accuracy of the DOV components. We have chosen the longest continuous QDaedalus measurement No. 47 at Pistahegy that lasted for 301 minutes. This data set contained 3746 measurement epochs and we selected 3085 epochs for processing. We note that to increase accuracy, our standard QDaedalus processing excludes each first measurement of a star from a series of continuous measurements to the same star. In this particular case we rejected the first from a series of 5 measurements.

The accuracy of inversion by the Danish method and Cauchy-Steiner weights for this particular measurement for DOV components ξ, η are shown on Figure 9. The horizontal axes of Figure 9 are scaled as $1/\sqrt{n}$, since if the law of large numbers is fulfilled, data points would be located along straight lines. We can see only partial fulfillment of this law, so other factors must contribute here. What is clearly seen, however, is the difference between the accuracy of the two methods. Inversion with Cauchy-Steiner weights consistently gives higher accuracies, and we can estimate that 0.05'' accuracy in this case can be reached after about 300 epochs or 30 minutes of measurement time.

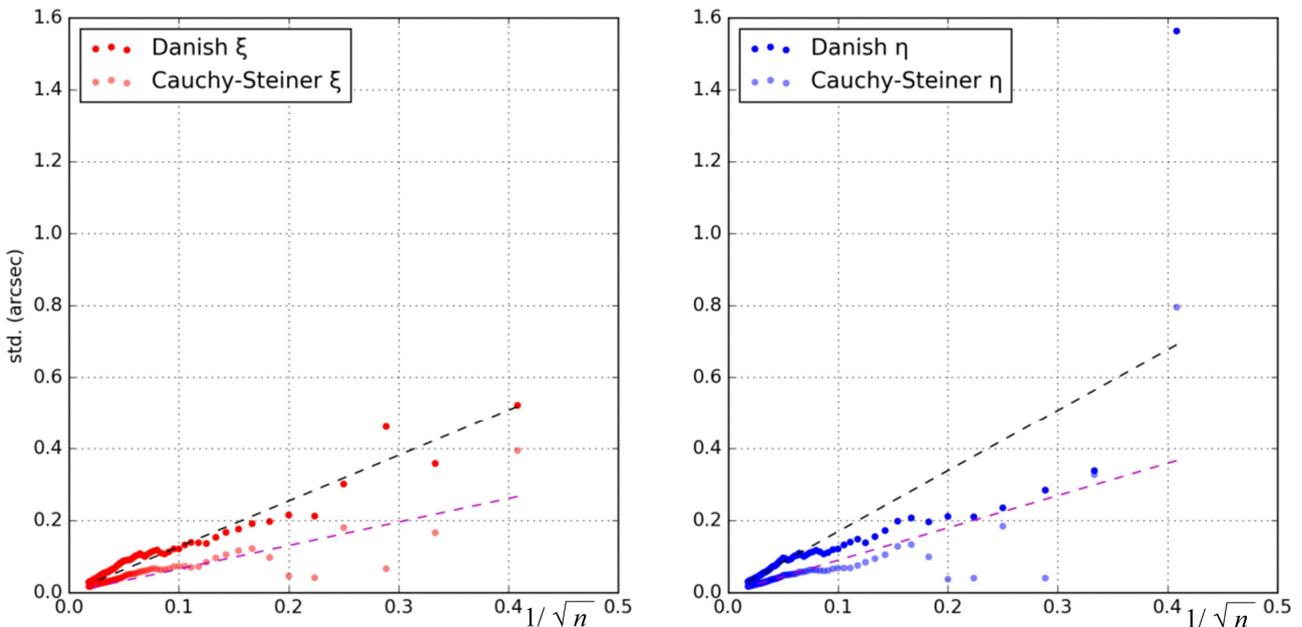


Figure 9

Accuracy of DOV components ξ, η determined by inversion as a function of the number of measurement epochs for measurement No. 47 at Pistahegy. If the law of large numbers were fulfilled, points would have to be located along a straight line. Best-fitting lines are also shown

6. CONCLUSIONS

We have compared two robust and resistant methods for the inversion of QDaedalus zenith direction determination. M-inversion using Cauchy-Steiner weights [13] yielded consistently better inversion results in terms of accuracy and correlation norm based on more than 70 independent measurement series. This is probably due to the higher statistical efficiency of M-inversion in case of a non-Gaussian distribution of residuals compared with the robust E-estimation with Danish method commonly used in geodetic data processing. By analyzing the accuracy of deflection of the vertical components we can state that 15-30 minutes of measurement time by QDaedalus is probably enough to reach DOV accuracy below 0.1".

7. LIST OF SYMBOLS

Symbol	Description	Unit
CDF	cumulative distribution function	-
DOV	deflection of the vertical, its components are: ζ, η	arcsec ["]
ECDF	empirical cumulative distribution function	-
ICRS	International Celestial Reference System	-
ITRS	International Terrestrial Reference System	-
MFV	most frequent value	-
K-S	Kolmogorov-Smirnov	-
PDF	probability distribution function	-
α, δ	equatorial coordinates (right ascension and declination)	°
δ_{refr}	refraction correction of vertical angles	°
\underline{B}	first Jacobian matrix of the implicit measurement function	-
ε	dihesion of residuals	arcsec ["], pixel
\underline{g}	model function (implicit)	-
\underline{G}	second Jacobian matrix of the implicit measurement function	-
ζ, η	N-S and E-W components of deflection of the vertical	arcsec ["]
\vec{e}	measurement residual vector	arcsec ["], pixel
\vec{e}'	projected measurement residual vector	arcsec ["], pixel
i_z	index error of the total station	arcsec ["]
$\vec{\lambda}$	vector of Lagrange multipliers	-
\underline{m}	parameter vector	°
ℓ, z	horizontal and vertical angles of the pointing axis of the telescope	°
ℓ^*, z^*	horizontal and vertical angles of a star	°
x_0, y_0	position of the image of the pointing axis of the telescope	pixel
x, y	position of the image of a star on the CCD sensor	pixel
φ, λ	geodetic latitude and longitude	°
Φ, A	astronomical latitude and longitude	°
\underline{W}	weight matrix	-
\underline{W}'	matrix of Cauchy-Steiner weights	-
ω	orientation correction	°

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