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EXPERIENCES OF QDAEDALUS MEASUREMENTS

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Abstract: QDaedalus is an automated, computer-controlled astro-geodetic measurement system developed at ETH Zurich. This paper reports results of three years of deflection of the vertical (DOV) measurements with the QDaedalus system at Pistahegy, located in the SE part of Budapest. We analyzed inversion residuals for the angles measured by the total station and found a horizontal angle bias for each new set of star observations. By incorporating the estimation of this bias in the least-squares Danish robust inversion procedure we reported a 25% improvement in terms of standard deviation of the DOV components. On the other hand, inversion with Cauchy-Steiner weights by using bias estimation gave worse results by 5–10% in terms of standard deviation of DOV, and the most frequent value procedure with Cauchy–Steiner weights is still the best on average by 19%. We also analyzed the measurement residual time series by using ensemble averaging. We found that stationarity no longer holds for the residuals obtained with horizontal circle bias estimation.

Keywords: *QDaedalus, astro geodetic measurements, deflection of the vertical, Danish robust inversion, Cauchy–Steiner weights, non-uniform time series.*

INTRODUCTION

Accurate determination of the direction of the gravity vector is an important task in geodesy and for certain engineering projects. Although GNSS systems give accurate positions, they cannot provide physical heights. These heights are related to equipotential surfaces of gravity and to their normal vector, the local zenith direction. This direction is usually referenced to an ellipsoidal normal vector and deviations of the zenith direction from the ellipsoidal normal vector must be determined in North-South and East-West directions. These are the components (ξ, η) of the deflection of the vertical (DOV), and can be determined by astrogeodetic methods.

Recent advances in instrumentation and computer technology have made it possible to accurately (up to 0.01″) determine DOV in the field by dedicated instruments (digital zenith cameras) or modern robotic total stations modified for the purpose; QDaedalus is such a system [1], [4].

Since integrity of the system and data processing are both critical for high accuracy DOV determination, we think these goals are best served by repeated measurements at the same site. First, a site with known DOV values gives an immediate check on the validity of the output of the system. Second, repeated measurements in a controlled environment give ample material for analyzing different factors contributing to the improvement of data processing methodologies, for analyzing environmental factors, and so on.

Our present work reports our experiences with the QDaedalus system based on three years of nearly continuous measurements at our reference site at Pistahegy (located in the SE part of Budapest). We report improved calibration of the instrument, and data processing issues.

In Section 2 a quick overview of the QDaedalus system is given. Section 3 presents the status of DOV measurements at Pistahegy site. In the next Section we present an improved data processing methodology. In Section 5 we analyze time series of measurements and in Section 6 some conclusions are drawn.

THE QDAEDALUS SYSTEM

The QDaedalus system is a computer-controlled, GNSS-assisted automatic measuring system, which can be used mainly for high-precision determination of the local vertical direction (deflection of the vertical) [1–4]. The main system component is an adapted Leica TCA 1800 robotic total station. The eyepiece of the instrument is replaced with a CCD sensor. The GNSS receiver provides accurate timing of star observations and measures WGS84 coordinates. The external computer controls the robotic total station via the QDaedalus software *(Figure 1)*. The software maintains a measurement database and calculates deflection of the vertical components.

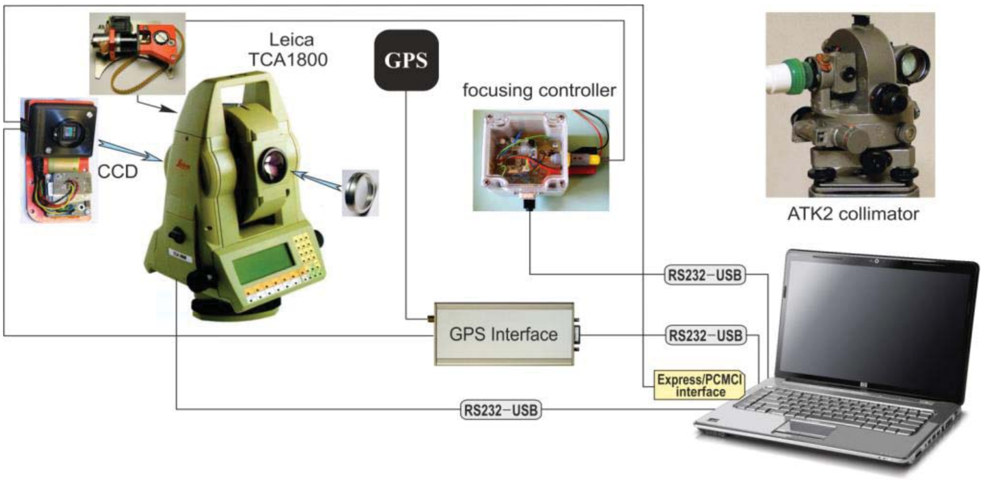

Figure 1. *Schematic construction of the QDaedalus system*

We used QDaedalus for high-precision determination of the local vertical direction (also known as deflection of the vertical). DOV components can be computed from geodetic coordinates of the local vertical and ellipsoidal normal [5, 6]:

$$\xi = \Phi - \varphi , \qquad (1)$$

$$\eta = \left(\Lambda - \lambda \right) \cos \varphi . \tag{2}$$

GNSS measurements provide geodetic latitude and longitude φ, $\lambda$with respect to the WGS84 ellipsoid, whereas astrogeodetic observations by QDaedalus system provide astronomical latitude and longitude Φ, Λbased on known celestial equatorial coordinates *(δ, α)* of measured stars [5].

MEASUREMENTS BY THE QDAEDALUS SYSTEM

For the precise determination of deflection of the vertical, a very long sequence was measured at the point *Pistahegy.*The location of the measurements are shown in *Figure 2*, the WGS84 coordinates of point Pistahegy areφ = 47˚24'53.8112" and λ = 19˚08'17.8948". From the autumn of 2015 until the end of 2018 we performed 180 measurements taken on 83 nights at this point in different seasons, in the most diverse meteorological, temperature and refraction conditions.

At the start of our measurements, the calibration of the CCD sensor and the inaccuracy of the computer clock caused various problems. At the beginning of a measurement, the most important step is to calibrate the instrument. In this step it is necessary to establish a connection between the readings in the horizontal and vertical circle of the total station and the coordinate system of the CCD sensor. The calibration must always be performed in situations when the CCD sensor is fixed on a new position on the total station, is removed, changed its fixing, or the outside temperature was changed significantly. For astronomical measurements, the parallax of the total station must be set to infinity, so the calibration must be performed in this position as the CCD sensor can only produce sharp images from infinite objects. In the beginning, we used motionless LED diodes at a distance of one or two hundred meters from the total station, but at night at this distance it was very difficult to handle the LED diodes, and light sources even at hundreds of meters distance did not produce a sharp image on the CCD sensor. We tried to use the star of α Ursa Min. (Polaris) for calibration, but it was not suitable for accurate calibration, because Polaris is not exactly in the direction of the rotation axis of the Earth, and this causes a small movement of Polaris. Thus we had to look for a solution that is suitable for simple and accurate calibration at night in field conditions. We solved the problem using a collimator. Our collimator is a special optical system that produces parallel beams of light from the object located at the eyepiece's focal point, similar to the way parallel beams of light from infinite distant objects (stars) arrive in our instrument. Our collimator planned for calibration measurements is a modified ATK-2 astronomical instrument (shown in the upper right corner of *Figure 1*, or on the tripod on the left of *Figure 2*), the parallax of which is adjusted fixedly onto infinity. The object suitable for calibration in the ATK-2 instrument is the reticule cross at the focal point, and a special LED was constructed to illuminate the reticule plate.

Figure 2. *Night measurement by the Qdaedalus system at the point* Pistahegy

Following the solution of the calibration and computer clock problem, after the first 77 measurements the accuracy of QDaedalus measurements improved significantly.

From the 140th measurement, another problem occurred in our measurements: the N-S components (ξ) of DOV values began to increase. We could not find the problem until the 170th measurement, and then it turned out that the parameters of the Leica TCA 1800 total station were constantly changing. After repairing and calibrating the Leica TCA 1800 total station, from the 171st measurement the Qdaedalus system measures acceptable DOV values again. This experience underlies the need for regular instrument integrity checks and calibration when high accuracy is required.

Figure 3 gives the results of all 180 Qdaedalus measurements of DOV at the Pistahegy station. The results of the Danish method and the Cauchy–Steiner weights [1] are presented.

The results of problem-free measurement series 78 to 139 and 171 to 180 are located in the hatched area in the middle of *Figure 3*; these points are marked by red dots and the serial number of each measurement is given next to the dot. The DOV values marked by blue stars computed from the first 77 measurements are separated in the lower part of the figure, and the values marked by black reverse triangles computed from the series of 140–170 Qdaedalus measurements can be found in the upper part of the figure.

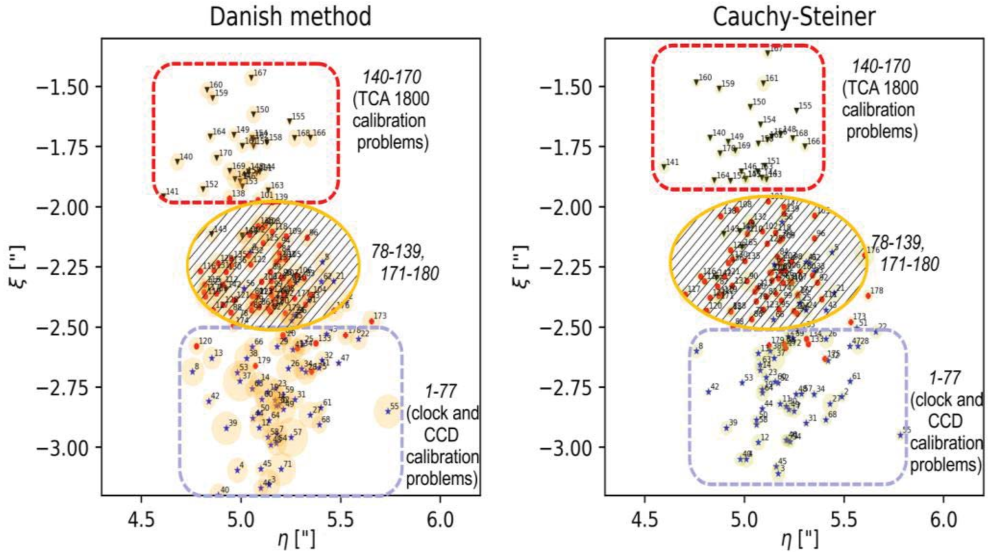

Figure 3. *DOV inversion results of 180 QDaedalus measurement series by the Danish method vs. Cauchy–Steiner weights at Pistahegy Station. Numbers of series are given on the plots. Ellipses indicate estimated inversion errors. Median of ξ is –2.39"/–2.37", median of η is 5.14"/5.15" for Danish/Cauchy–Steiner methods, respectively*

INVERSION PROCEDURE WITH HORIZONTAL ANGLE BIAS ESTIMATION

In our previous paper [1] we discussed in detail two different data processing methodologies. The first was a standard robust least-squares inversion for the unknown parameters (the Danish method). The second one was a novel robust inversion using Cauchy–Steiner weights. On average, inversion with Cauchy–Steiner weights gave a smaller standard deviation of DOV components by 35% based on the statistics of 180 measurements.

Our standard measurement scheme involves at least 10 subsequent observations of a selected star. Repeated observations increase accuracy and decrease measurement time, since it is quite time-consuming for the robotic total station to set the theodolite axis to a new star separated by a wide angle from its present position. On the other hand, it is desirable to measure as many stars as possible with a good distribution to get accurate results. Therefore it is not desirable to take too many observations on a particular star. We also found by experimentation that it is best to drop the first measurement of the selected star to increase accuracy. Readings of the compensator of the total station were quite uncertain for the first measurements and this lead to larger angle measurement errors. Inversion residuals were always much larger for the first measurements of a selected star.

In addition to this, by analyzing inversion residuals we found another issue. To illustrate this point, let us take a typical example. *Figure 4* shows the distribution of inversion residuals of horizontal and vertical (zenith) angles for measurement No. 176 with the Danish method. Whereas the distribution of zenith angle residuals is quite random, the horizontal angle residuals arrange themselves differently. It is obvious from the figure that the scatter of residuals between *different* stars (denoted with different colors) is much bigger than for the *same* star. The same is true for the residuals of the Cauchy–Steiner method; the horizontal angle residuals, however, are smaller in this case – they lie in the ±8" range.

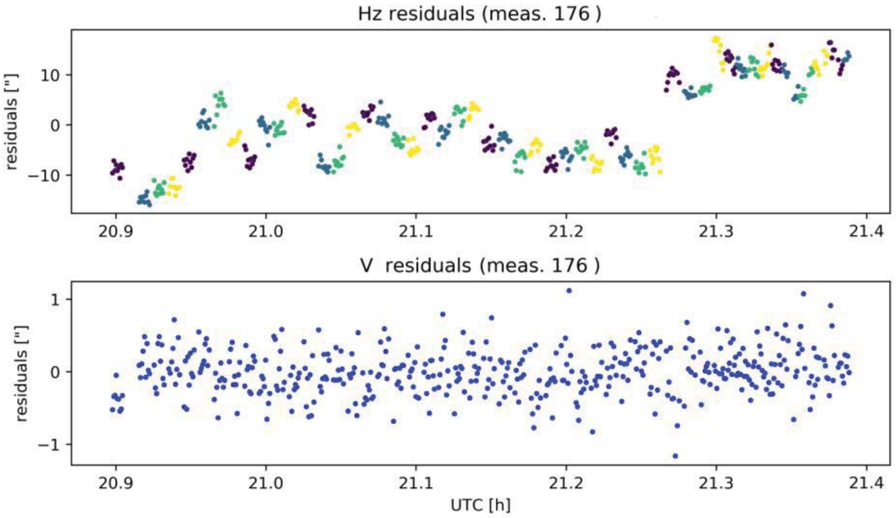

Figure 4. *Distribution of inversion residuals of horizontal and vertical (zenith) angles for measurement No. 176 with the Danish method. Whereas zenith angle residuals distribute randomly, horizontal angle residuals distribute differently. In the upper subfigure different colors denote sets of observations of different stars, each set containing 9 measurements. It is obvious that the scatter of residuals between different stars is much larger than for the same star*

How can this phenomenon be explained? Although the details of the setting out procedure of the telescope axis of the Leica total station are unknown to us, we think it logically consists of two steps. First a quick setting of the axis takes place to roughly the desired direction. Next this direction is corrected by fine adjustments on the axis. This step must involve the reading of the dual-axis compensator of the instrument. If the liquid in the compensator is not sufficiently settled, its suggested direction correction may be off by several seconds of arc. This may lead to a bias in the horizontal direction. Vertical angles are much less affected: the compensator is attached to the alidade of the instrument, and vertical telescope motion is independent of the alidade motion.

For subsequent observations to the same star only fine adjustments of the telescope axis are needed as star positions vary in the meantime. As time passes, the compensator liquid will become sufficiently settled and the scatter of horizontal measurements will remain small with respect to the first horizontal angle measurement.

To fix this error and improve accuracy, unknown biases were introduced as additional parameters of the inversion for each set of horizontal angle observations to the same star. This process in the following will be called *horizontal angle bias estimation* (HABE). We included additional bias terms ℓ_k in the model equations for horizontal angles for each set of star observations

$$\left(\ell+\ell_k-\ell^*\right)\sin z^* - a_{11}\left(x-x_0\right)-a_{12}\left(y-y_0\right)=0\,, \tag{3}$$

where ℓ denotes reading to the pointing axis of the telescope, ℓ^*, z^* are the computed Hz and V directions of the star, x, y denote the position of the star's image on the CCD sensor and x_0, y_0, a_{11} and a_{12} are calibration constants [1].

If $n_S$is the number of observed stars, the size of the $\underline{\underline{\mathbf{G}}}^{\mathbf{gh}}$ matrix of the inversion equation system $\underline{\underline{\mathbf{G}}}^{\mathbf{gh}}\overrightarrow{\mathbf{m}}=\vec{\mathbf{g}}_{\mathbf{0}}$ is increased by n_S. The number of unknowns in the vector $\overrightarrow{\mathbf{m}}$ and size of the constant vector $\vec{\mathbf{g}}_{\mathbf{0}}$ are increased by the same amount. This modified inversion procedure was applied with both Danish and Cauchy–Steiner weights as discussed in [1].

Figure 5 shows distribution of inversion residuals of horizontal angles for measurement No. 176 with the Danish method. Now the distribution is much more random and the residuals are much smaller than those shown on the upper part of *Figure 4*. The average improvement reaches 25% in terms of standard deviation of DOV accuracy based upon the statistics of 99 measurements. On the other hand, inversion with Cauchy–Steiner weights by using Hz bias estimation gave worse results, with a 5–10% increase in terms of standard deviation of DOV. The most frequent value procedure with Cauchy–Steiner weights is still the best on average by 19% compared with the Danish method applying HABE.

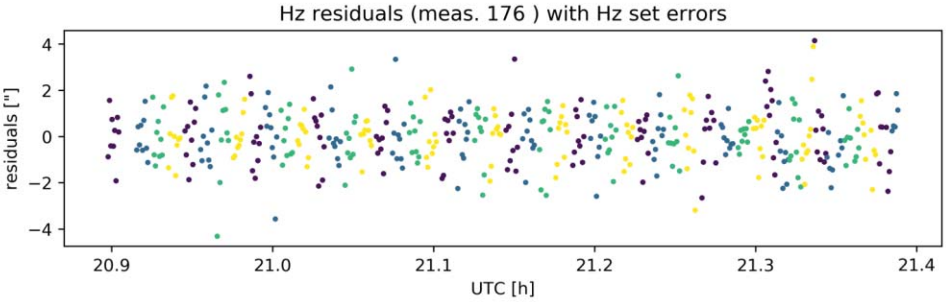

Figure 5. *Distribution of inversion residuals of horizontal angles for measurement No. 176 with the Danish method applying HABE. We see that by fixing biases during the inversion, horizontal angle residuals now distribute randomly. Different colors denote sets of observations to different stars*

TIME SERIES ANALYSIS OF MEASUREMENT RESIDUALS

Measurements with QDaedalus are essentially time-dependent observations. Precise time tags are provided for each measurement by the system via the built-in GPS component. In order to get insight on the measurement process we analyzed the inversion residuals as time series.

Supposing that the process is essentially the same for different stars during the observation we will assume that these can be treated as different realizations of the same stochastic process. Time tags of the horizontal and vertical measurements are used as the time variable. If we subtract the known model (horizontal and vertical) angles for each measurement epoch, we get values of a stochastic process. Different realizations are the series of observation residuals for each star. In the following processing it is assumed that we have sets of irregularly sampled data $x_i = x(t_i)$ at sampling times t_i $(i = 0, ..., N-1)$. There were 4,825 such sets (realizations) for the last 99 measurements at Pistahegy. We computed statistics (averages, standard deviations, auto- and cross-correlation functions) of the underlying stochastic process by ensemble averaging [7].

The ensemble autocorrelation function (ACF) is a function of the time t and of the delay τ:

$$R(t,\tau) = E\left[\left\{x(t) - E\left(x(t)\right)\right\}\left\{x(t+\tau) - E\left(x(t+\tau)\right)\right\}\right], \tag{4}$$

where $E\{\ \}$ denotes ensemble averaging. The formula is similar for the cross-correlation function (CCF) of $x(t)$ and $y(t)$; we replace $x(t)$ by $y(t)$ in one place in the product on the right side of *Equation (4)*.

The sampling of the process is non-uniform. In *Figure 6* we see how uneven the distribution of sampling times is. To deal with such non-uniformity we calculated slot statistics for $\Delta t = 1\,\text{s}$ long time slots. The slotting technique derives the autocorrelation function $R(t, \tau)$ at discrete time lags $t_i = i\,\Delta t,\ \tau_k = k\,\Delta t$ by averaging the products of all data pairs x_i and x_j falling into bins of the width Δt.

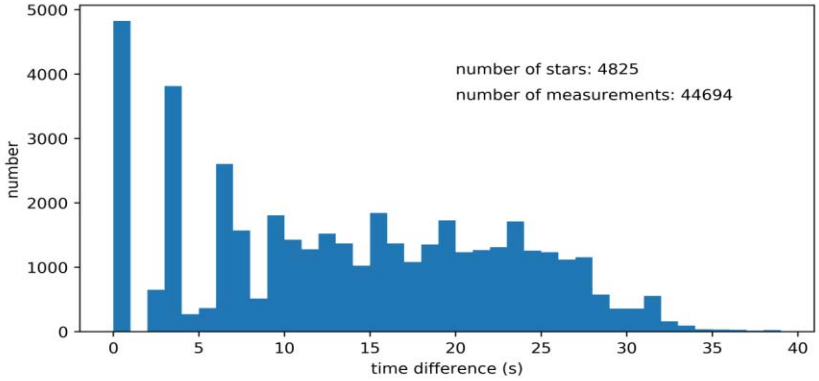

Figure 6. *Uneven distribution of sampling times of measurements 82–180 to 4,825 stars observed at Pistahegy Station. Bin size is 1 s. The second bin is empty, since no measurement is possible within such a short time interval after the first one with QDaedalus*

Mean and standard deviation by ensemble averaging are shown in *Figure 7* in case of the Danish method with and without horizontal angle bias estimation. We can see a difference here. Horizontal angle bias estimation resulted in significantly bigger standard deviations for certain epochs (4 s, 11 s, 18 s, etc.). The same is true for processing with Cauchy–Steiner weights. If the stochastic process is stationary, the statistics of the process are independent of time. Since this is not the case for HABE, we cannot assume stationarity, whereas assumption of stationarity can be accepted as a working hypothesis for results without HABE.

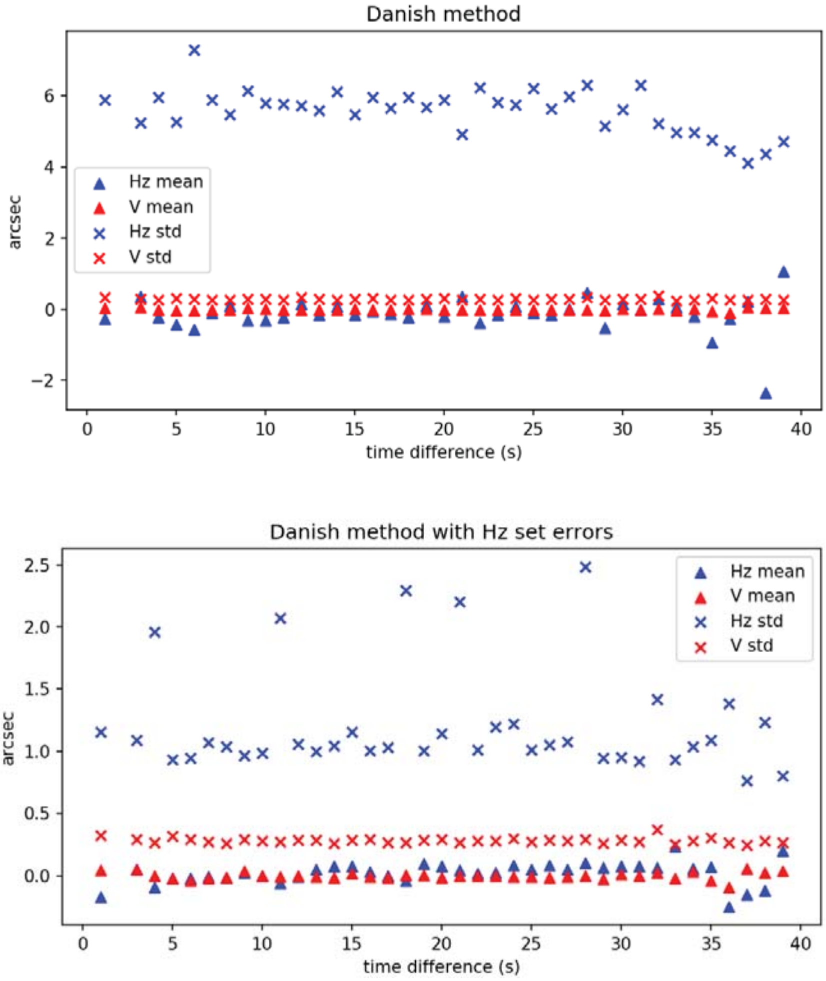

Figure 7. *In the upper subfigure ensemble averaged statistics of angle measurement residuals are shown for the Danish method. The time axis shows time differences with respect to the first measurement in each series of star observations. In the lower subfigure the same statistics are shown for the Danish method with HABE. If a process is stationary, these statistics must be time independent*

The auto- and cross-correlation functions (ACF and CCF) of horizontal and vertical angle measurement residuals are shown in *Figure 8* for different epochs for the Danish method with HABE. We again see significantly bigger autocorrelations for zero lags at the same epochs 4 s, 11 s and 18 s for horizontal angles (non-stationarity). The ACF and CCF values are very small (below 10^{-17}) for non-zero lags in the range of ±8 s. These statistics support the processing of measurements at different epochs as independent observations.

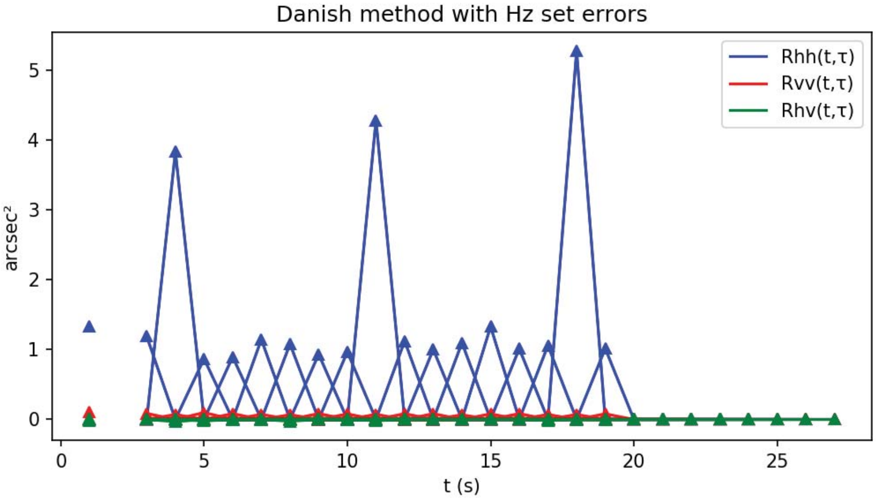

Figure 8. *The auto- and cross-correlation functions (ACF and CCF) of horizontal and vertical angle measurement residuals for the Danish method with HABE. We observe bigger ACF for certain epochs for horizontal angles. The ACF and CCF values were calculated with 1 s slots for lags in the range of ±8 s*

CONCLUSIONS AND PLANS

This paper reports results of three years of deflection of the vertical (DOV) measurements with the QDaedalus system. The results show that QDaedalus is a system capable of precise and economical DOV determination. It was also shown that integrity of the system components is critical for high precision. Possible instrument miscalibration and the lack of proper computer clock setting and calibration can hinder accurate DOV determination.

Analysis of measurement residuals revealed certain ways to improve data processing and inversion results. Especially such an analysis led us to a horizontal angle bias estimation procedure as part of the inversion that improved accuracy of DOV determination with the Danish robust least-squares procedure by 25% in terms of standard deviation. We also analyzed the measurement residuals as time series by using ensemble averaging. We found that stationarity no longer holds for the residuals obtained with horizontal angle bias estimation.

Our plans with the QDaedalus system are the following. First of all, we would like to continue DOV monitoring at the Pistahegy site. Accuracy of any measurement system critically depends on the detection and elimination of possible systematic errors. In this respect, the simple affine calibration model must be critically examined. Since QDaedalus measurement is essentially time dependent, processing with Kalman filtering would be an obvious alternative to that currently used. This might pose a challenge, however, since the QDaedalus measurement model consists of implicit nonlinear equations. To deal with such situations the implicit UKF procedure recently proposed by [8] might be applicable.

ACKNOWLEDGEMENT

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LIST OF SYMBOLS

Symbol	**Description**	**Unit**
ACF	autocorrelation function	arcsec^2 [$''^2$]
CCD	charge coupled device	
CCF	crosscorrelation function	arcsec^2 [$''^2$]
DOV	deflection of the vertical, its components are: ξ, η	arcsec [$''$]
GNSS	Global Navigation Satellite System	–
HABE	inversion with horizontal angle bias estimation	–
MFV	most frequent value	–
UKF	Unscented Kalman Filter	–
WGS84	World Geodetic System 1984	–
a_{11}, a_{12}	affine calibration parameters	°/pixel
$E\{\ \}$	ensemble averaging	–
$\underline{\underline{\mathbf{G}}}^{\mathbf{gh}}$	matrix of the system of inversion equations	–
$\vec{\mathbf{m}}$	parameter vector	°
ℓ, z	horizontal and vertical angles of the pointing axis of the telescope	°
ℓ^*, z^*	horizontal and vertical angles of a star	°
ℓ_k	horizontal angle bias	°
$R(t, \tau)$	autocorrelation function	varies
x_0, y_0	position of the image of the pointing axis of the telescope	pixel
x, y	position of the image of a star on the CCD sensor	pixel
α, δ	equatorial coordinates (right ascension and declination)	°
δ_{refr}	refraction correction of vertical angles	°
φ, λ	geodetic latitude and longitude	°
Φ, Λ	astronomical latitude and longitude	°
ξ, η	N-S and E-W components of deflection of the vertical	arcsec [$''$]

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