REDUCING THE MEASUREMENT TIME OF THE TORSION BALANCE

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Abstract

The main problem of torsion balance measurements is the long damping time however it is possible to significantly reduce it by modern technology. The damping curve can be precisely determined by CCD sensors as well as computerized data collection and evaluation. The first part of this curve makes it possible at least theoretically to estimate the final position of the arm at rest. A finite element solution of a fluid dynamics model based on Navier-Stokes equations is presented here to solve the problem.

Keywords: Eötvös torsion balance, damping time, CCD sensor, Navier-Stokes equations, CFD, finite elements

1 Introduction

Starting in the fifties of last century gravimetry has gradually replaced difficult and time-consuming terrestrial gradiometry in gravity exploration. There is, however, a renewed interest about terrestrial gradiometry by the Eötvös' torsion balance, mainly for geodesy (Völgyesi et al. 2009a, 2009b). With the advance of techology, however we have the possiblity to reduce long measurement time (40 min for each azimuth) and to automate the measurement process. The goal of this paper is to study the physical equations of motion of the balance in view of the new claims of reducing measurement time. We constantly keep in mind, however, that this shorter measurement time should still meet the accuracy requirements.

2 Observations for studying torsion balance damping

A laboratory has been developed to make various tests and measurements by the Eötvös torsion balance in the Technical University of Budapest. These tests were made with our own AUTERBAL (Automatic Eötvös-Rybar Balance) equipment. Cameras with CCD sensors were mounted on the reading arms for automatic readout as it is shown on Fig. 1. A sample CCD image can be seen on Fig. 2. Control of cameras and taking shots was computer-driven with the necessary software developed under the Linux operating system. Since with these cameras several shots and readings per second for a long period of time can be taken, a new perspective is ahead of us to observe hitherto unknown phenomena. It became feasible, for example, to study damping characteristics of the device with far more detail and accuracy than it was previously possible.

Four shots per second were taken in several 40-50 min. long records in all azimuths to study damping of the balance. Time resolution was increased up to 12 shots per second (i.e. 0.08 s sampling period) to examine the finest details of the damping curve, whereas two 24-hour long records were taken to study possibly long-period kinetics of the balance.

Processing of these records was accomplished by our computer software specially developed for the purpose. Computerized evaluation of records comprised of examining relative shifts of sequential images along an axis parallel to the scale by 2D cross-correlation techinque.

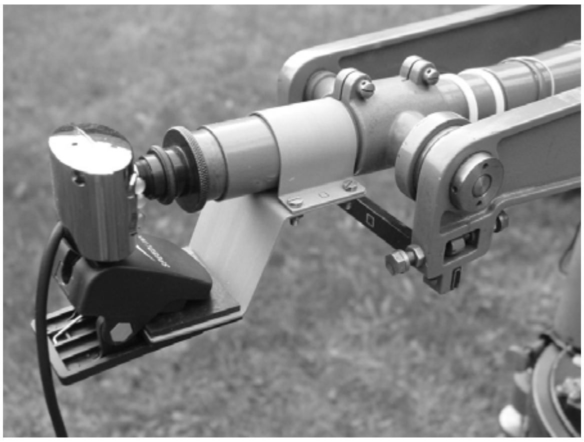

Fig. 1. Mounting of a CCD camera for automatized readout

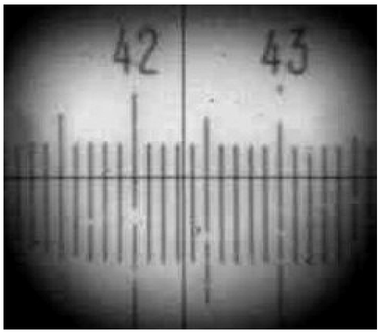

Fig. 2. Reading by CCD camera

A representative damping curve is shown on Fig. 3., which was processed by the technique described above.

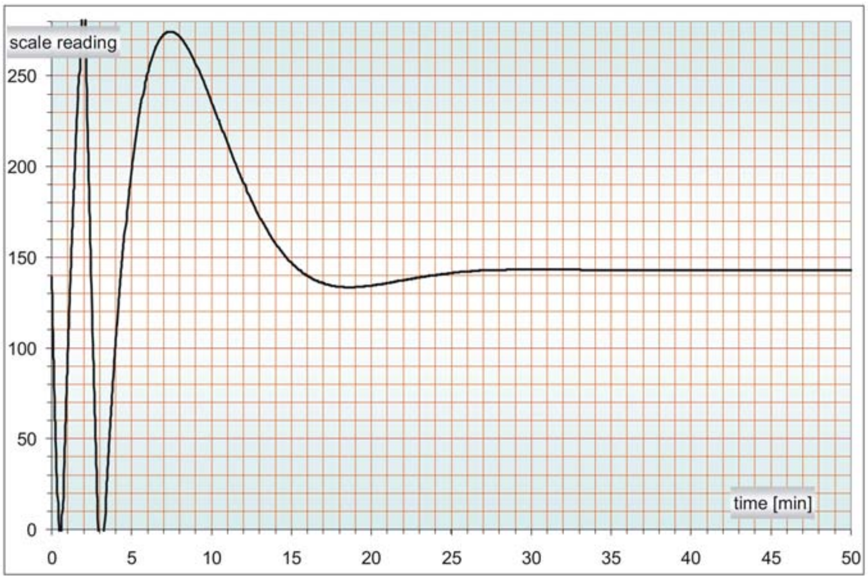

Fig. 3. A representative damping curve

3 A 7-DOF model of the Eötvös torsion balance

The generally adopted physical model of the Eötvös torsion balance is a simplified one degree-of-freedom (DOF) model, including torsional-mode oscillations only (Selényi, 1953). It is reasonable, however, that a sufficiently detailed mechanical model has to be developed for interpreting and evaluating measurements with an enhanced torsion balance equipped with a CCD sensor. For this end we made use of Lagrangian mechanics (Landau and Lifshitz, 1976).

A dynamical system can be characterized by the (generally time-dependent) Lagrangian $L(q, \dot{q}, t)$ of the system, where q and $\dot{q}$ denote generalized coordinates and velocities, respectively.

If the dynamical system has s degrees of freedom, its motion is described by the following Lagrange equations of the second kind

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)-\frac{\partial L}{\partial q_i}=0\,, \qquad (i=1,2,...,s) \tag{1}$$

where $L = T(q,\dot{q}) - V(q)$ is the difference of kinetic and potential energy.

When our system is conservative (there are no dissipative forces) and standard (i.e. holonomic and constraint forces do no work), these equations completely characterize motion of the dynamical system. This model is suitable for studying the motion of the balance, since on the one hand in a first approximation dissipative forces can be neglected, and on the other hand the detailed examination of damping, where these dissipative forces are important, will be done separately.

Let us consider Fig. 4 to write Lagrange equations of the Eötvös balance. The balance can be decomposed into three masses (torque-arm with mass m' and the two attached masses m), and can be characterized by 7 DOF's as the following set of 7 generalized coordinates completely describes any possible configuration of the system (the configuration space has 7 dimensions):

$$q=(\alpha,\delta;\beta,\gamma;\theta,\eta;\varepsilon)$$

The upper end of torsion wire is mounted at point B whereas its length and torsion coefficient are denoted by s and τ, respectively.

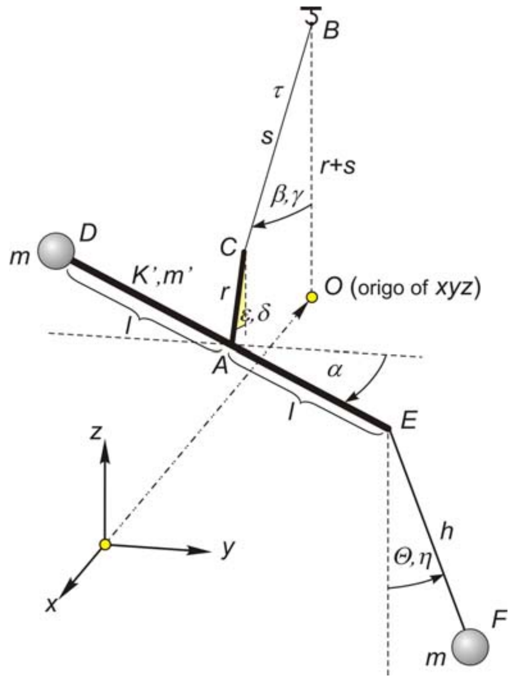

Fig. 4. Dynamical model of Eötvös torsion balance

The lower end of torsion wire BC at point C is attached to the rigid $DE + AC$ arm with mass m' and moment of inertia K'. The upper mass m has its centre of mass (COM) placed at point D at a distance ℓ from point A (centre of the arm). The lower mass m has its COM at point F, hanging on a massless wire of length h below point E.

To define generalized coordinates q which characterize any configuration of the balance let us introduce a right-handed Cartesian system xyz (Fig. 4). Its origin O is aligned vertically (z) with point B at a distance of $r + s$, where r denotes the distance between points A and C. Let axis x be perpendicular at rest to *DE* arm, whereas let axis *y* point perpendiculary in the direction of the lower mass *m*. Direction of *BC* with respect to axis *z* are given by angles β and γ with respect to *x* and *y*; direction of *CA* with respect to axis *z* are described by angles ε és δ in directions of *x* and *y*. Angular position of the torque-arm *AE* with respect to axis *y* in directions of *x* and *z* are characterized by angles α and δ. Finally θ and η give angular position of *EF* in directions of *x* and *y* with respect to *z.*

To express kinetic and potential energies, cartesian coordinates (x_D, y_D, z_D), (x_F, y_F, z_F) for the positions of the masses and that of the COM *P* of the torque-arm (x_P, y_P, z_P) are required. Moreover, the velocity components (v_{Dx}, v_{Dy}, v_{Dz}), (v_{Fx}, v_{Fy}, v_{Fz}), (v_{Px}, v_{Py}, v_{Pz}) are required as well. Potential energy *V* of the system on the one hand comes from the potential energies of the above three bodies, on the other hand from the potential energy of the twisted torsion wire. If we set potential energy of the system at rest to zero and denote by *g* the acceleration of gravity at *P*, we can write

$$V = \tfrac{1}{2}\tau\alpha^2 + mgz_D + mg(z_F + h) + m' g z_P \,. \tag{2}$$

Total kinetic energy T of the system is the sum of translational kinetic and rotational energies, expressible by linear and angular velocities with respect to the coordinate axes. Let K' , I_x' , I_y' . denote moments of inertia about COM *P* of the torque-arm with respect to axis parallel with axes *z*, *x*, *y*, respectively. Hence total kinetic energy of the system is

$$T = \tfrac{1}{2}K'\dot{\alpha}^2 + \tfrac{1}{2}I_x'\dot{\delta}^2 + \tfrac{1}{2}I_y'\dot{\varepsilon}^2 + \tfrac{1}{2}mv_D^2 + \tfrac{1}{2}mv_F^2 + \tfrac{1}{2}m'v_P^2 \,, \tag{3}$$

where $v_D = \sqrt{v_{Dx}^2 + v_{Dy}^2 + v_{Dz}^2}$, $v_F = \sqrt{v_{Fx}^2 + v_{Fy}^2 + v_{Fz}^2}$, $v_P = \sqrt{v_{Px}^2 + v_{Py}^2 + v_{Pz}^2}$, and dot means time derivative as usual.

Lagrange equations of the enhanced dynamical model of the Eötvös torsion balance are thus obtained by substituting Eqs. (2) and (3) into Eq. (1). By using the Lagrangian $L(q,\dot{q},t)$ the modal analysis, i.e. natural frequencies and mode shapes of this multiple-DOF mechanical system can be obtained. Omitting here the details we mention that there will be no coupling between two groups of generalized coordinates $\boldsymbol{q}_1 = [\alpha,\beta,\varepsilon,\theta]^T$ and $\boldsymbol{q}_2 = [\gamma,\delta,\eta]^T$. These two groups describe independent transversal (cross-arm) and longitudinal (along-arm) components of the motion, respectively.

Following the method described in Landau and Lifshitz (1976) and omitting again the details, natural circular frequencies ω of the system can be found by solving the following generalized eigenvalue problems (where eigenvectors $\boldsymbol{A}$ represent the mode shapes of the system, i.e. the small amplitudes of motion of each DOF):

$$(\omega^2 \boldsymbol{M}_i - \boldsymbol{K}_i)\,\boldsymbol{A}_i = \boldsymbol{0} \,, \qquad i = 1, 2. \tag{4}$$

By multiplying Eqs. (4) with $\boldsymbol{K_i}^{-1}$ and denoting matrix $\boldsymbol{B_i} = \boldsymbol{K_i}^{-1}\boldsymbol{M_i}$ we arrive at the following two regular eigenvalue problems with the new eigenvalues $\gamma = 1/\omega^2$:

$$\boldsymbol{B}_i\,\boldsymbol{A}_i = \gamma\,\boldsymbol{A}_i \,, \qquad i = 1, 2. \tag{5}$$

A numerical solution needs computation or at least estimation of the parameters in Eqs. (5), which are physically related to the problem. With that natural frequencies $f_i = 1/(2\pi\sqrt{\gamma_i})$ can be computed from eigenvalues γ_i . Estimates of these parameters for the Auterbal-type balance as well as corresponding periods $T_i = 1/f_i$ can be found in Table 1.

Angular position α of the arm is what the CCD sensor captures in the first place. These are the transversal mode shapes T2, T3, T4 in Table 1. Mode shape T4 describes torsional oscillations, however we expect to discover also T2 and T3 mode shapes in the CCD record with sufficient sampling rate. To check this we made spectral analysis of a 5 min. long, nominally 12.5 Hz sampling rate record. To make the sampling rate of this time series exactly even, Akima spline interpolation in the time domain at regular 0.08 s interval points was performed. PSD estimation was done by the sine multitaper method of Riedel and Sidorenko (1995). Power spectral density of the time series is shown on Fig. 5.

Table 1. Estimated natural frequencies and periods of the Auterbal-type balance. Significant amplitude in α belongs only to mode shapes marked with an *.

Transversal mode shapes	f_i [Hz]	T_i [s]
i = 1 (T1)	26.044	0.0384
i = 2 (T2*)	3.1859	0.3139
i = 3 (T3*)	0.8560	1.1683
i = 4 (T4*)	0.00096853	1032.50
Longitudinal mode shapes	f_i [Hz]	T_i [s]
i = 1 (L1)	2.1754	0.4597
i = 2 (L2)	1.5848	0.6310
i = 3 (L3)	0.8054	1.2416

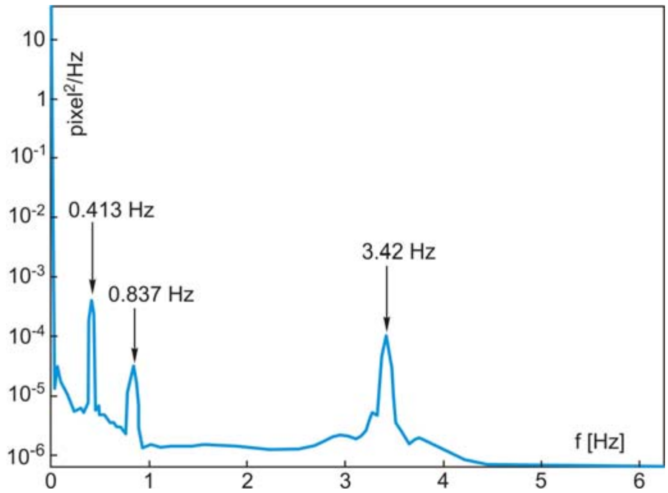

Fig. 5. PSD of time series captured by CCD sensor at 12.5 Hz sampling rate.

Three specific components can clearly be identified in the PSD. One with the highest frequency is at 3.42 Hz, which is by 7% higher than the theoretical value belonging to shape mode T2 (3.19 Hz). The component at 0.837 Hz deviates from the T3 theoretical value by only 2%. The 0.413 Hz component is interesting. Its frequency is approximately half that of the T3 mode shape. Our provisional idea to explain this component is that the assembly of two antiparallel balances makes it possible to couple somehow their motion through the structure of their suspension. Or, with a lower probability, this is an artifact of the applied CCD image processing and correlation technology. Anyhow, further elaboration of the image processing technique and of the dynamical model will hopefully be decisive on this question.

A practical consequence of this is the following. It is clear that our time series comprise oscillations of various frequencies due to the peculiarities of the mechanical system, which describes motion of the torque-arm. However, the only mode shape required to have an estimate on the home positon of the arm is T4. Hence it is desirable to remove all frequencies linked with other mode shapes from time series that can be considered as „noise" before making an estimate of the reading of the home position.

4 A viscous damping model of the balance

The simplest model of an arm oscillating in the chamber is a viscously damped torsional oscillation model. According to the theory of damped oscillations (Landau and Lifshitz, 1976) in the under-damped case of the arm the following function of 5 independent parameters describes the motion:

$$x = a_0 + a_1 e^{-a_2 t} \cos(a_3 t - a_4)\,, \tag{6}$$

where x and t denote displacement and time, respectively and a_0, ..., a_4 are parameters.

The most important parameter to be estimated is a_0 since it is the reading of the home position of the arm.

According to what was mentioned in the previous section, an IIR lowpass Butterworth filter was designed to remove frequencies above 0.4 Hz in the signal. The corner frequencies of the filter were 0.32 and 0.4 Hz. PSD of a lowpass filtered time series (Fig. 6) of the balance marked as [] (square), captured in the first azimuth illustrates nicely the effectiveness of the filtering process. The power of the filtered signal is reduced by 11 orders of magnitude for frequencies higher than 0.5 Hz.

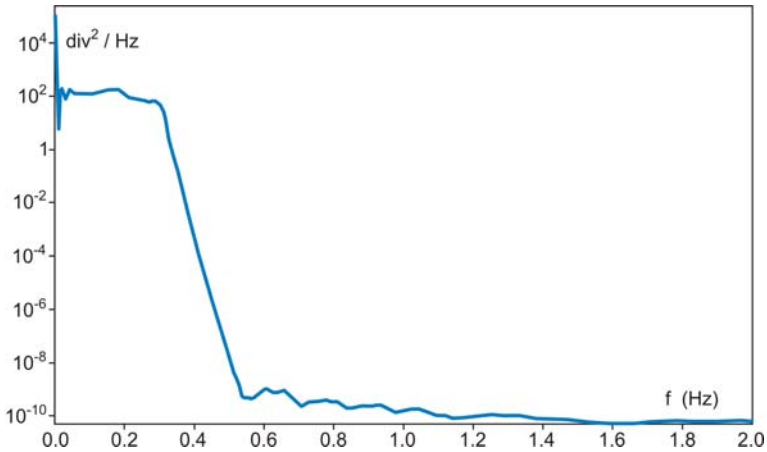

Fig. 6. PSD of time series of the balance marked as [] (square), captured in the first azimuth with 4 Hz sampling rate (div means scale division units)

This low-pass filtered time series was used to estimate home position by fitting an exponential damping model to it. Period of the shape mode T4 of about 1023 s is significantly biased by viscous damping.

When Eq. (6) is formulated as a conventional least-squares adjustment, the design equations for n data are

$$c_0 + c_1 e^{-a_2 t_i} \cos(a_3 t_i) + c_2 e^{-a_2 t_i} \sin(a_3 t_i) - x_i = 0\,, \qquad i = 1, \ldots, n.$$

These equations yield optimal least-squares estimation of the 5 parameters sought. Since the equations are non-linear for some of these parameters, an iteration until fixed-point convergence of the parameters is required.

For subsequent tests we have selected several subsets of the time series (measured for the [] balance in azimuth 1.) between time frames t_1 and t_2. Our main concern was the amount of bias between the estimated parameter a_0 and its correct value a_0 = 422.48 (scale division units). These biases are shown as a function of t_1 and t_2 in Table 2.

Table 2. reveals an optimum fit between 150 s and 600 s for this particular case. Naturally, additional tests are required to reach an overall best fit and to decide on optimal values of t_1 and t_2. These results, however, are encouraging and show that biases of the home position reading lower

than a couple of tenth of scale division units may be feasible by using only first 10 minutes of the measurement data.

5 Fluid dynamics model

More accurate modelling of the arm's motion is possible by fluid dynamics models. Numerical modelling is possible by e.g. finite elements using geometrical and physical parameters of moving bodies, casing and that of the damping fluid (air). A complicated solid-fluid (air) interaction arises inside the casing (chamber) of the balance, since motions of solid and fluid are strongly coupled and interdependent. These problems are termed „multi-phyisics problems" since both mechanical and fluid dynamics modelling are required for problem solving.

Air is a viscous media and at standard temperature and pressure (STP) up to 100 m/s flow velocities it can be treated as an uncompressible fluid. The motion of such fluids is governed by the following Navier-Stokes equation (Pozrikidis 2001, p. 282)

$$\frac{D\boldsymbol{v}}{Dt} = -\frac{1}{\rho}\nabla p + \nu\, \nabla^2 \boldsymbol{v} + \frac{1}{\rho}\boldsymbol{F} ,$$

where $\boldsymbol{v}$ is the flow velocity, p is the pressure, ρ is the fluid density, $\boldsymbol{F}$ represents body forces (per unit volume) acting on the fluid, D/Dt is the material derivative, and ν is the kinematic viscosity. The Navier-Stokes equation derives from conservation of momentum (Newton's second law) of viscous fluid flow. This equation was the basis of two-dimensional computational fluid dynamics (CFD) modelling of arm's motion and air flow inside the chamber.

To start, we fixed the geometrical and physical parameters of our model. Geometrical parameters and the computational mesh are shown in Fig. 7.

Physical parameters of the model (units are cm, g, hectoseconds):

- density of the air at STP: 0.00129,
- kinematic viscosity: 0.0182,
- torsion coefficient of the wire: 660,
- moment of inertia of the arm: 2400, homogeneously distributed.

Initial values:

- initial angular position of the arm: -0.02 rad,
- initial angular velocity of the arm: 0.

Penalty method proposed by Janela et al. (2005) was used for CFD modelling of the solid-fluid (arm-air) motions.

Number of the triangles of the finite element mesh was 15386, the number of vertices was 7829. Penalty parameter was fixed at the value of 10^{-6}.

Table 2. a_0 parameter bias for time series of the balance [] in azimuth 1. as a function of first and last time frames of the least-squares fit (in scale division units)

t_1 (s)	t_2 (s)	a_0 bias (div)
60	400	-7.18
100	400	-11.26
60	600	-2.80
120	600	-1.5
180	600	0.54
150	**600**	**-0.16**
150	720	0.42
150	660	0.79
150	420	-0.93

The initial velocity field was obtained by solving a Stokes problem. Time step was 20 s and 50 steps were computed.

For details of the penalty method the reader is referred to the paper by Janela et al. (2005) and the references therein. To summarize the method briefly, at each time step it is required to solve a system of two equations over the triangles of finite element mesh. In these equations division by the small penalty parameter enforces a nearly zero deformation constraint inside the domain belonging to the moving arm. This technique yields rigid body motion of the arm in the solid-fluid interaction CFD modelling. We have developed a 400-line program for the numerical solution in the high-level language of FreeFem++ sofware (http://www.freefem.org, 2011-01-25).

Simulation results of a successful test for modelling the arm's motion are seen on Fig. 8. A four-parameter least-squares fit to the simulated angular position data was obtained using Eq. (6) (where now the parameter a_0 is zero). Misfits to the viscous model described by Eq. (6) are of amplitude 10^{-4} rad (0.2 – 0.3 scale division) and quasi-periodic with a period of approximately 400 s. Misfits of a similar character were experienced after a viscous model fit to the CCD captured time series. This may be an indicative of the limits of the simple viscous model for high accuracy requirements.

Estimates can be obtained for increased period T of the oscillation and viscous damping parameter a_2. Numerically, we obtained 1027 s for T and 0.264 for a_2 instead of the correct (theoretical) values of 1198 s and 0.4, respectively. Also computational instabilities happened for particular values of the mesh density, viscosity and penalty parameter. Hence there is a lot of room for improvement of the CFD modelling, and we plan to use other methods, such that the distributed Lagrange multiplier/fictitious domain method by Glowinski et al. (1998). Another possibility is to use the oomph-lib library especially developed for true 3D multi-physics problems (Heil and Hazel 2006).

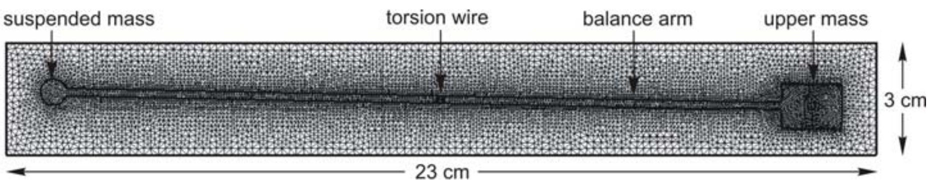

Fig. 7. Main geometrical parameters of the chamber and computational mesh for two-dimensional CFD modelling of the Eötvös torsion balance (upper view)

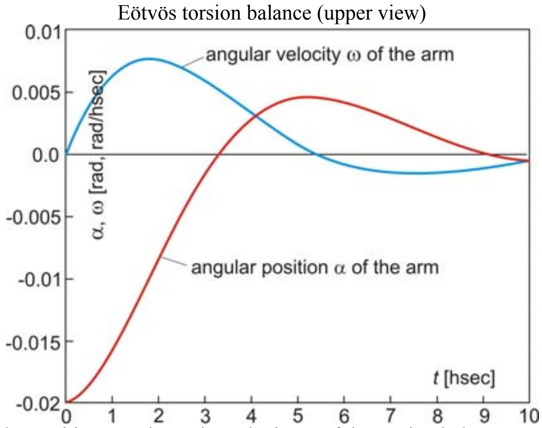

Fig. 8. Time-dependent angular position α and angular velocity ω of the torsion balance arm obtained by CFD modelling (time t is measured in hectoseconds, 1 hs = 100 s)

6 Summary

This paper demonstrated the possibility of reducing the long measurement time of Eötvös torsion balance by using advanced technology. Observations captured by CCD sensors were processed and a detailed dynamical model of the balance was developed. Our study showed that these achievements may make it possible in the near future to cut down measurement time in each azimuth from 40 to 10 minutes to obtain accurate enough estimate of the home position of the balance. Our fit bias data in Table 2. are suggestive of this, indicating a bias that is under 1 scale division after processing 10 minutes of filtered time series. Interestingly, this is approximately the time according to Figs. 3 and 8 when angular velocity of the arm firstly becomes zero. Naturally, further innovation and research are required to develop a method that can be routinely applied in everyday field work with the Eötvös torsion balance.

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Hivatkozások

Glowinski R, Pan T W, Hesla T I, Joseph D D (1998): A distributed Lagrange multiplier/fictitious domain method for particulate flow. Int. J. of multiphase flow, 25, 755-794.

Heil M, Hazel A L (2006): oomph-lib – An Object-Oriented Multi-Physics Finite-Element Library. In: Fluid-Structure Interaction, Editors: M. Schafer und H.-J. Bungartz. Springer (Lecture Notes on Computational Science and Engineering), 19-49.

Janela J, Lefebvre A, Maury B (2005): A penalty method for the simulation of fluid-rigid body interaction, ESAIM Proceedings, 14, 201-212.

L.D. Landau, E.M. Lifshitz (1976): Mechanics. Vol. 1 (3rd ed.). Butterworth-Heinemann.

Pozrikidis C (2001): Fluid dynamics: theory, computation and numerical simulation. Kluwer Academic Publishers. 658 o.

Riedel K S, Sidorenko A (1995): Adaptive Smoothing of the Log-Spectrum with Multiple Tapering. IEEE Transactions on Signal Processing, 43, 188-195.

Selényi P (1953): Eötvös Loránd összegyűjtött munkái, Akadémiai Kiadó, 386 o.

Storn R, Price K (1997): Differential Evolution – A Simple and Efficient Heuristic for Global Optimization over Continuous Spaces. Journal of Global Optimization, 11, 341-359.

Völgyesi L, Égető Cs, Laky S, Tóth Gy, Ultmann Z (2009a): Eötvös-inga felújítása és tesztmérések a budapesti Mátyás-hegyi-barlangban. Geomatikai Közlemények, XII, 71-82.

Völgyesi L, Csapó G, Laky S, Tóth Gy, Ultmann Z (2009b): Közel fél évszázados szünet után ismét Eötvös-inga mérések Magyarországon. Geodézia és Kartográfia, LXI, 11. 71-82.

Tóth Gy, Völgyesi L, Laky S (2014): Reducing the Measurement Time of the Torsion Balance. Earth on the Edge: Science for a Sustainable Planet (Eds: C. Rizos, P. Willis) International Association of Geodesy Symposia Vol. 139, Springer-Verlag; Berlin, Heidelberg, ISBN: 978-3-642-37221-6. pp 341-347.