HUNGARIAN CONTRIBUTION TO THE RESEARCH ON NUMERICAL THEORIES AND SOLUTIONS IN MATHEMATICAL GEODESY – IAG INTER-COMMISSION COMMITTEE

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The theoretical and practical background of the similarity transformation together with the simultaneous estimation of local geoid undulations is presented in Bányai (2011). The mean features of the traditional network adjustment on the local ellipsoids are summarized and the different Hungarian networks and known geoid solutions are shortly described as the basic data of the test computations. The eigenvalue and eigenvector decomposition revealed that the seven parameter similarity transformation cannot be applied together with the simultaneous local geoid estimation because the rotations about the X and Y axes significantly destroy the condition of the normal equations. However, the replacement of the rotations about the X, Y and Z axes by the rotation about the ellipsoidal normal of the datum point can provide a very well-conditioned solution, which takes into account the special role of the datum point of the astro-geodetic network adjustment. Based on the unit weights of the input data an optimal adjustment strategy is demonstrated from a computational point of view, where the five transformation parameters can be estimated together with a very large number of local geoid undulations. The geoid has to be known in the global reference system. The geoid unknowns describe only the relative position of this known geoid with respect to the local reference system. The application of the available and the simultaneously estimated local geoid solutions proved that neglecting local geoid heights has a most significant impact on the scale parameter, while it has no significant effects on the horizontal residuals from the statistical point of view. The small scale difference (1 ppm) and the small rotation (−0.5 arc sec) about the datum point and its ellipsoidal normal of the Hungarian local system with respect to the global GPS system demonstrate the high quality of the traditional measurements as well.

Classical numerical integration methods were tested for determining the orbits of most recent Low Earth Orbiter (LEO) satellites. In general, numerical integration techniques for orbit determination are commonly used to fill the gap between two discrete, observed epochs. In Somodi and Földváry (2011, 2012) orbits were determined using the EGM96 gravity model by the Euler, Runge-Kutta, Bulirsch-Stoer and Adams-Moulton numerical integration techniques among others. These analyses were performed for LEO satellite GOCE and for one medium altitude GPS satellites. The orbits were integrated under different assumptions on the roughness of the force model, considering effects of elasticity, high order gravity and non-static Earth generated accelerations on the orbits.

Subdivision surfaces are widely used in computer aided design and animation, but rarely in geoinformatics. In the paper of Czimber (2011) the most important subdivision methods are discussed and a new procedure is presented, which is able to control the interpolation or approximation by points and the adaptive subdivision of the triangles in geoinformation systems.

The exterior orientation of sensors (e.g. camera-systems) is one of the basic tasks of the photogrammetry. The parameters for exterior orientation can be determined from the mathematical equations between the image coordinates and the corresponding object or ground coordinates. The mathematical models for this problem have been available since decades; huge program packages utilize the methods which have proved to be successful in practice. In Závoti and Fritsch (2011) a new

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alternative solution is given. This paper proposes an alternative solution, which does not use iteration and approximate data. The equations in this work are in coherence with the photogrammetric theory of exterior orientation; the only difference is in the mathematical solution. This kind of mathematical treatment of the problem can be considered as novelty.

In Bányai (2012) the exact least-square line fit with errors in both coordinates is investigated together with the approximate solution based on the formalism of the linear Gauss-Helmert model or the unified adjustment approach of the classical textbooks. The similarities and the differences are described in details. In spite of the small differences the exact solution is preferable and the calculations are simpler. This paper does not deal with the errors-in-variables (EIV) models solved by the total least-squares (TLS) principle, since the exact line fit solution is used to validate this general approach, which is basically designed to solve more sophisticated nonlinear tasks. In the most general case the fit of Person's data with York's weights is iteratively solved starting with the arbitrary zero initial value of the slope. The test computation with different but systematically chosen weights proved that in special cases – e.g. the weighted least-square sum of the distances between the data points and the estimated line is minimised – there is no need for iterations at all. It is shown that the methods described by Závoti (2012b) are special cases of the general exact solutions. The simple linear estimation of variance-covariance matrix of the exact solution is also demonstrated. The importance of the stochastic models coupled with exact solution is also demonstrated.

In Paláncz (2012) the algebraic solution of the geometric model of photogrammetric exterior orientation is presented by a system of multivariate polynomial equations. Employing Dixon resultant, the determination of the roots of this system can be reduced to the computation of the roots of a single variable polynomial of fourth order. In this case the Dixon matrix does not have full rank; therefore the standard Nakos-Williams algorithm cannot compute the resultant of the polynomial system.

The laws of nature in general and the relations and laws, particularly in geodesy, can be expressed in most cases by nonlinear equations, which are generally solved by transforming them to linear form and applying iteration. The process of bringing the equations to linear form implies negligence and approximations. In certain cases it is possible to obtain exact, correct solutions for nonlinear problems. In Závoti (2012a) rotation matrix parameters are introduced and used for the solutions of 2D and 3D similarity transformations. This method involves no iteration, and it does not require the transformation of equations into linear form. The scale parameter is determined in both cases by solving a polynomial equation of second degree. This solution is already known, but this derivation is worth to be considered because of its simple nature.

In the last ten years the application of computer algebra systems to special basic tasks has become one of the most rapidly developing branches of geodetic research. The conventional methods for solving problems involve approximation and iteration; and because of the lack of proper innovation, this is the general approach even today. Computer algebra systems have led to the construction of models, which give exact, analytical solutions. In many cases these models can't be applied, because increasing the number of the data leads to a combinatorial explosion, that is, a general solution can't be computed even with today's modern computers. The paper of Závoti (2012b) describes some basic geodetic tasks, for which new, stable solutions already exist.

The demand for integrated adjustment of different geodetic observables arose from practical reasons. The popular basic concept of the seventies and sixties was reconsidered in Bányai (2013). A new procedure was developed for the adjustment of precise geodetic observables, by which the astro-gravimetric data – geoid undulations and deflections of the vertical – can be taken into account in different ways. New "quasi-linear" observation equations were introduced for geodetic total station measurement, which have a more convenient numerical advantage with respect to traditional approach. The method is tested and demonstrated by field measurements. Rotational residuals and additional parameters – scale differences and antenna phase centre offsets – can be used to handle the outliers of GNSS baseline components aided by proper statistical tests. The common application of GNSS baselines and levelled height differences proved to be an efficient tool to improve the height component of local 3D networks. If the deflections of the vertical are comparable to the accuracy of geodetic total station measurements the integrated adjustment is preferable.

Datum transformation has been widely used in geodesy and a number of different algorithms have been known and applied. However, many of them are based on the assumption of small rotations, and linearization is needed in order to derive the datum transformation parameters. In Papp (2013) the concept of quaternions is described to represent the rotation and scale parameters in Bursa-Wolf geodetic transformation model. The main advantage of this algorithm is that it can be applied in case of arbitrary size rotation; it does not need linearization and iteration for computation of the datum transformation parameters for a non-linear transformation model.

The Dirichlet distribution is one of the most important multivariate probability distributions with wide range of applications in various areas of statistics, probabilistic modelling, engineering and geosciences. The paper of Monhor (2013) is an application-driven short and simplified introduction to the fundamental issues of the Dirichlet distribution and gives some useful representations of bounds on the Dirichlet distribution function. A new polynomial representation for the bivariate Dirichlet distribution is established. The potential possibility of geodetic and geophysical applications of the Dirichlet distribution is briefly described within the framework of recent developments and trends of statistical science and applied probability.

In the paper of Paláncz et al. (2013) the Pareto optimality method is applied to the parameter estimation of the Gauss-Helmert weighted 2D similarity transformation assuming that there are measurement errors and/or modeling inconsistencies. In some cases of parametric modeling, the residuals to be minimized can be expressed in different forms resulting in different values for the estimated parameters. Sometimes these objectives may compete in the Pareto sense, namely a small change in the parameters can result in an increase in one of the objectives on the one hand, and a decrease of another objective on the other hand. In this study, the Pareto optimality approach was employed to find the optimal trade-off solution between the conflicting objectives and the results compared to those from ordinary least squares (OLS), total least squares (TLS) techniques and the least geometric mean deviation (LGMD) approach. The results indicate that the Pareto optimality can be considered as their generalization since the Pareto optimal solution produces a set of optimal parameters represented by the Pareto-set containing the solutions of these techniques (error models). From the Pareto-set, a single optimal solution can be selected on the basis of the decision maker's criteria. The application of Pareto optimality needs nonlinear multi-objective optimization, which can be easily achieved concurrently via hybrid genetic algorithms built-in engineering software systems such as Matlab. A real-word problem is investigated to illustrate the effectiveness of this approach.

In Földváry and Csapó (2014) the role and the suitability of point data for describing analytical surfaces in surveying and geodesy is discussed. Within the frame of this study no overall analysis is presented, but rather the relevance of the problem is emphasized through an actual case study; i.e. the reliability of describing the gravity field by gravimetric networks. All in all, the conclusion is that as long as the points are not capturing precisely the extremes with suitable point distribution, the surface may fail. In the case of several quantities, such as gravity, the extremes cannot be located uniquely based purely on observed data. In such cases contour lines of the quantity of interest derived on the point-wise data may drastically differ from the real shape of the surface, as it is experienced by the unrealistically high alteration of two different epochs of the Hungarian gravity network (based on notably different point distribution), MGH-50 and MGH-2000.

There are continuous observations, which are carried out with varying sampling rates, however, their processing needs high resolution. To solve the problem different approximation and interpolation methods were investigated in Kalmár (2014). It was experienced that the trigonometric polynomials can be applied very efficiently to interpolate geomagnetic baseline measurements and to derive the measurement errors. This procedure can be easily implemented even in Excel spreadsheet.

In Mohamed et al (2014) integrated baseline adjustment and similarity transformation method is proposed as an alternative strategy for the regional size Cairo Network to estimate intra-plate deformations using GPS observations. The proposed method is demonstrated to estimate coordinate changes, global rotations and scale parameters in one computational step. The proposed method is used to investigate the significance of the impact of global plate motions on regional crustal movement network. Simulated data of the regional Cairo network is used for this evaluation. The estimated plate motions, simulated scale bias (due to miss-modelling of troposphere effect on GPS data) and baseline noise proved that the impact of plate motions have to be taken into account in the case of Cairo network if the investigation period is near or larger than ten years.

In Bácsatyai (2014) the description of the HUGAPRO program system is given, which was created for transformation between all the projections and reference systems used in Hungary for practical, educational and research purposes. It can be applied to compute the transformation parameters, projection reductions, standard deviations and maximum discrepancies in any combination between two chosen projection, reference or auxiliary systems. Between arbitrary projections the parameters of 7 parameter similarity or polynomial transformations can be determined and can be used to carry out the necessary transformations.

The paper of Závoti (2014) presents an important theoretical problem of geodesy: we are looking for a mathematical dependency between two spatial coordinate systems utilizing common pairs of points whose coordinates are given in both systems. In geodesy and photogrammetry the most often used procedure to move from one coordinate system to the other is the 3D, 7 parameter (Helmert) transformation. Up to recent times this task was solved either by iteration, or by applying the Bursa-Wolf model. Producers of GPS/GNSS receivers install these algorithms in their systems to achieve a quick processing of data. But nowadays algebraic methods of mathematics give closed form solutions of this problem, which require high level computer technology background. In everyday usage, the closed form solutions are much simpler and have a higher precision than earlier procedures and thus it can be predicted that these new solutions will find their place in the practice.

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