Hungary's New Gravity Base Network (MGH-2000) and It's Connection to the European Unified Gravity Net

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Abstract. The modernization of the Hungarian Gravity Base Network was carried out in the framework of the establishment of the unified gravity network of the Central-European countries. The objective of former networks and the necessity of development are reviewed. The scale of the new network is guaranteed in SI system by the numerous absolute gravity measurements carried out of late years. The applied observation methods, data processing and adjustment procedures are presented. The results of the comparison of *Unified European Gravity Network'94* (UEGN-94), Czech, Slovakian and Austrian networks with the Hungarian network (MGH-2000) are discussed.

Keywords. Gravity, network, absolute–, relative measurements, adjustment of gravity network

1 Introduction

One of the major consequences of the political changes in Hungary was the abolition of the secrecy of gravimetric information during the early 1990s. This made it possible for researchers to take part in the international professional projects.

The US National Imagery and Mapping Agency (NIMA, formerly DMA) began to increase the accuracy of WGS-84 reference ellipsoid in 1991 and substantially helped Hungary establish both a Hungarian national Military GPS Network (KGPSH) and an absolute gravimetric base network, Ádám et al. (1994).

2 Antecedents

Hungary's first gravity network (MGH-50) covering the entire territory of the country was established by Loránd Eötvös Geophysical Institute (ELGI) during the early 1950s. The measurements were carried out by a Heiland gravimeter, Renner and Szilárd (1959).

During the 1960s the 1st order network was reobserved and extended by geodetic type Sharpe gravimeters, Csapó and Sárhidai (1990/A).

In the 1970s a new second order gravity network was established. The height of the points was determined by leveling. The gravimetric measuring of the network was completed with two Sharpe CG–2 and a geodetic type LaCoste–Romberg (LCR) instruments between 1980 and 1989. The common adjustment of the measurement carried out in 1971 and between 1980 and 1989, as well as compiling a catalogue of points was implemented in 1991. The standard deviation of the adjusted network of MGH-80 is ± 16 μGal (1 μGal = $1 \times 10^{-8} ms^{-2}$) Csapó and Sárhidai (1990/B).

The objective to create an international gravimetric network, was defined in the co-operation projects of the geodetic surveys of the East Central European countries in the mid-1960s. The concept of this network was rather similar to the existing ones. The observations needed for the establishment of this common network had continuously been carried out by the experts of ELGI in bilateral and multilateral forms of co-operation since 1972, Csapó et al. (1994).

In 1992 and 1993 gravimetric measurements were carried out in the form of interconnecting measurements between Austria and Hungary within the framework of bilateral co-operation. This activity consisted of relative and absolute measurements. Relative measurements were carried out with 4–5 LCR gravimeters, whereas absolute measurements were carried out with a JILAG–6 absolute gravimeter, Csapó et al. (1993).

The implementation of the Unified European Gravity Network (UEGN) was commenced in 1993 in the East Central European countries under the auspices of the joint plan of the International Gravity Commission (IGC) and Geodetic and Geophysical Working Group of NATO (GGWG) in the form of international co-operation. Absolute measurements

were carried out in Hungary between 1993 and 1995 with AXIS FG5 No.107 gravimeter of NIMA and with JILAG–6 equipment of BEV.
In 1994 the tying of absolute stations with relative gravimeters was commenced with four LCR–G instruments transported by car. Hungary's absolute stations are situated at an average distance of 100 to 120 km, so 1st order and 2nd order points are used as tie points.

3 The structure of MGH-2000

3.1 The zero order network

The use of such network is meant to ensure the scale of the national base network as well as checking the stability of gravity by repeated observations. The zero order network consists of 15 absolute stations (6400km^2/point). Their location is given in Fig. 1, including foreign absolute stations near the borders as well. These points were placed at the ground level of significant buildings whose survival and accessibility seem to be ensured for a long time (manor-houses, mansions, etc.). 120 by 120 by 100 cm concrete blocks monumentation was implemented at floor level. A brass bolt was fixed to the middle of the upper level of the block indicating the height above sea level according to the Baltic system. The points were tied to two or three points of the national leveling network, providing $\pm$ 5 mm accuracy.

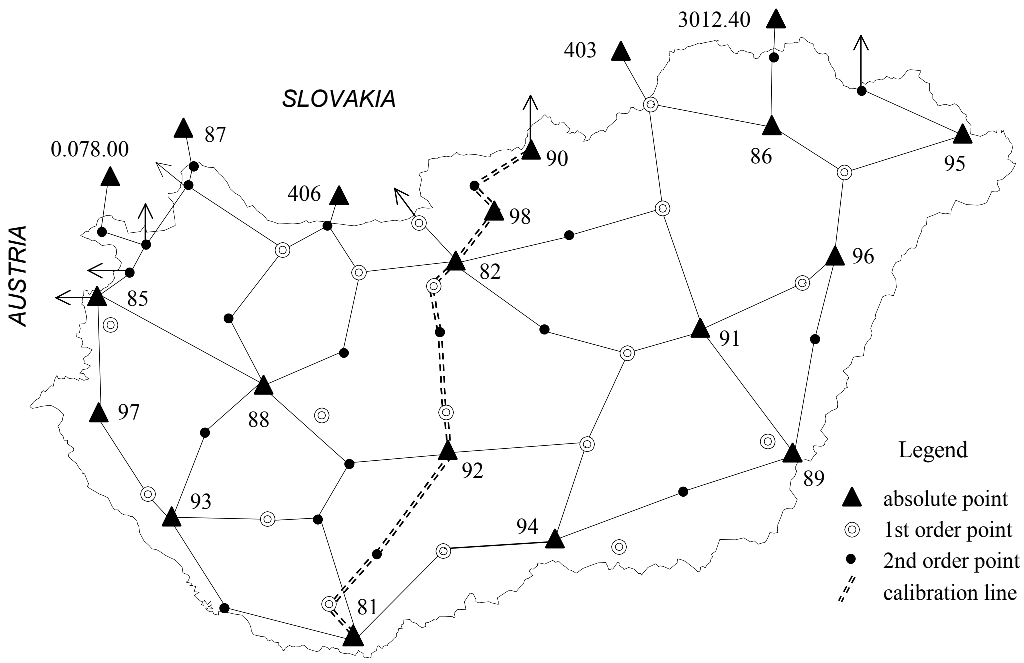

Fig. 1 The Hungarian part of UEGN

The geographical coordinates of the stations was determined on the basis of 1:10000 topographic maps with $\pm$1 second confidence limit. Gravity values relative to the reference heights of absolute gravimeters was determined with LCR gravimeters allowing 1,5–3 μGal confidence limit. The station established in Budapest is of extraordinary importance, because measurements have repeatedly been carried out on it with absolute gravimeters of different type in every two or three years since 1980. At most stations repeated measurements were carried out in the past three or four years as well. Each station has got at least one so called "excenter point" which is monumented with a concrete block outside the building. The relative confidence limit of its g value is not worse than 5 μGal. The most important parameters of the zero order network can be seen in *Tab. 1*.

Table 1. Parameters of the absolute stations of MGH-2000

	Station	Latitude (° ′ ″)	Longitude (° ′ ″)	Height (m)	Year of the first/last observation	Gravimeter
81	SIKLÓS	45-51-10	18-07-55	128.489	1978 / 1995.	GABL / JILAG-6
82	BUDAPEST	47-32-00	19-01-00	201.563	1980 / 2000.	GABL / AXIS FG5
85	KŐSZEG	47-23-24	16-32-33	284.461	1980 / 1993.	GABL / JILAG-6
86	SZERENCS	48-09-56	21-12-21	111.243	1980 / 1993.	GABL / JILAG-6
88	NAGYVÁZSONY	46-59-23	17-42-00	241.085	1993 / 1997	AXIS FG5 / JILAG-6
89	GYULA	46-38-42	21-17-14	89.053	1987 / 1995.	GABL / JILAG-6
90	SZÉCSÉNY	48-05-07	19-31-08	166.888	1993 / 1996.	AXIS FG5
91	KENDERES	47-14-54	20-40-37	83.450	1993.	AXIS FG5
92	MADOCSA	46-41-19	18-57-40	93.758	1994.	AXIS FG5
93	IHAROSBERÉNY	46-21-48	17-06-17	203.898	1994.	AXIS FG5
94	ÖTTÖMÖS	46-17-04	19-40-47	124.042	1994.	AXIS FG5
95	TARPA	48-06-14	22-31-40	110.778	1995 / 1996.	AXIS FG5
96	DEBRECEN	47-33-30	21-37-26	124.132	1996 / 2001.	IMGC / JILAG-6
97	ZALALÖVŐ	46-50-51	16-35-13	190.816	1997.	JILAG-6
98	PENC	47-47-20	19-16-52	245.668	1998 / 2000.	ZZG/AXIS FG5

3.2 First order network

The 19 points included in *Fig. 1* are nearly the same as the bases of MGH–80 placed at airports, Csapó and Sárhidai (1990/B). The distance varies between 50 to 70 km and the density of points is 4400 km^2/point. The determination of geographical coordinates of the points was similar to the methods described in section 4.1. Leveling between the individual points and the national leveling base points was done with ±1 – ±10 mm confidence limit.

3.3 Second order network

Second order network points were established by ELGI in the 1970s. The distance between these points is 10 to 15 km in hilly areas, whereas it ranges between 15 to 25 km in the plains. The average density of points is 250km^2/point. We have replaced a couple of dozens of points, which were destructed during the last twenty years and have integrated them into MGH-80. The new network contains 386 2nd order points. In Fig. *1*. only those points are presented which are members of the point catalogue of UEGN-2000.

4 National gravimetric calibration line

Its present form had been developed since 1969; its present status was accepted in 1985. Formerly it served as the scale of the Potsdam Gravity System. There are five absolute stations within the 210 mGal range of the line – the highest δg value is 250 mGal between the base points of the country (*Fig. 1*). δg values between the points were previously determined with Askania Gs–12, GAG–2, Sharpe, Worden, then LCR gravimeter groups. The vertical gradients of the points were determined with a group of 3-4 LCR instruments with an accuracy of 4–7 μGal, Csapó (1987). The relative accuracy of each point is 8–12 μGal.

4.1 Calibration of gravimeters

ELGI carried out measurements twice a year on the calibration line. ELGI has also built laboratory calibration equipment, Csapó and Szatmári (1995) for the calibration of LCR gravimeters with feed back electronics. With the help of this equipment the possible time variations of the calibration factor of LCR gravimeters can be studied as well, Meurers (1994).

5 Data processing

5.1 Absolute measurements

Table 1 shows that absolute measurements were carried out with five different types of instruments in Hungary in the last two decades. It was noted that some differences appeared over the course of the data processing as well as at the application of corrections. Therefore, it was necessary to reproc-

ess all observations according to a unified approach.
Instrument corrections were accepted given by the observers.
The calculation of tide correction was based on 505 tidal waves. The parameters were compiled according to the data of Pecný (Czech Republic) tide registration station, Holub et al. (1986). Ocean tide was disregarded.
Correction due polar motion. The observed gravity is corrected for changes in the centrifugal acceleration due to the variation of the distance of the earth's rotational axis from the gravity station. The following formula was used in the calculations:

$$\delta g_{pm} = 1.164\omega^2 R \sin 2\varphi \left(x\cos\lambda - y\sin\lambda\right)[ms^{-2}]$$

(ω = $7.292 \cdot 10^{-5}$ rad/s^2 angular velocity of the Earth's rotation; R = $6.371 \cdot 10^6$ m mean radius of the Earth; φ, λ geographical coordinates of the absolute station).
The value of correction in μGal:

$$\delta g_{pm} = -19.1 \sin 2\varphi \left(x\cos\lambda - y\sin\lambda\right)$$

The actual values for x and y can be found in the Annals of the International Earth Rotation Service (IERS).
Correction due to the changing atmospheric masses. Only the local part of the atmospheric effect was taken into consideration. The relation given by the standard of DIN 5450 of 1968 and with the following empirical coefficient:

c = 0.30 μGal /hPa .

The local part of the correction compensates for 80% of the total atmospheric effect.
The reduction of gravity value to the benchmark. In the adjustment the gravity values relevant to benchmarks were used. Therefore, the results of the absolute measurements that are related to a certain height depending on the type of the equipment (350–1300 mm) were referred to the benchmarks. Presently the relative measurements are carried out in four different heights above the bench mark and g is described as a second order function of height h :

$$g^{(h)} = ah^2 + bh + c$$

In this case the vertical gradient *(VG)* is obtained by the differentiation of the above function:

$$(VG) = \frac{dg^{(h)}}{dh} = 2ah + b$$

The vertical gradients were determined by a group of LCR gravimeters.

5.2 Relative measurements

The computer stored field records were processed in daily sections and by gravimeters. Processing steps:
- Conversion the readings to mGal
- Correction calculations (tidal, height, barometric and periodic error corrections)
- The calculation of corrected relative gravity values
- Drift calculations
- Calculations of drift corrected relative gravity values
- Δg calculation
- Error calculations.

6 Adjustment of measurements

The observed data were adjusted by the least square method as a constrained network. The fixed points of the network were the latest g values of the absolute gravity stations, Csapó and Sárhidai (1990/B). The mean value of observed gravity difference (Δg) observed in A–B–A–B–A system by one gravimeter (which means the average of the four observed difference) was taken as one individual measurement.
Taking into account the large number of unknowns and the problems of numerical stability of adjustment computations, the matrix orthogonalization method was used for practical adjustments, Völgyesi (1979, 1980, 2001). The base principle of matrix orthogonalization method can be demonstrated by the hyper-matrix transformation:

$$\begin{bmatrix} \underset{(n,r)}{\widetilde{\mathbf{A}}} & \underset{(n,1)}{\widetilde{\mathbf{l}}} \\ \underset{(r,r)}{\mathbf{E}} & \underset{(r,1)}{\mathbf{0}} \end{bmatrix} \rightarrow \begin{bmatrix} \underset{(n,r)}{\widetilde{\mathbf{W}}} & \underset{(n,1)}{\widetilde{\mathbf{v}}} \\ \underset{(r,r)}{\mathbf{G}^{-1}} & \underset{(r,1)}{\mathbf{x}} \end{bmatrix}$$

where

$$\underset{(n,r)}{\widetilde{\mathbf{A}}} = \underset{(n,n)}{\mathbf{P}^{1/2}} \underset{(n,r)}{\mathbf{A}} ,$$

$$\underset{(n,1)}{\widetilde{\mathbf{l}}} = \underset{(n,n)}{\mathbf{P}^{1/2}} \underset{(n,1)}{\mathbf{l}} ,$$

$\mathbf{A}$ is the coefficient matrix of the observation equations, $\mathbf{l}$ is the vector of absolute terms, $\mathbf{P}$ is the weight matrix, $\mathbf{E}$ is a unit matrix, $\mathbf{0}$ is a zero vector; $\mathbf{W}$ is a matrix having orthogonal columns, and $\mathbf{G}^{-1}$ is an upper triangular matrix, n is number of equation, r is number of unknowns, Völgyesi (1979, 1980).

This matrix transformation directly yields the wanted unknowns x_i in place of vector $\mathbf{x}$, variances and covariances of unknowns x_i are comprised in weight coefficient matrix

$$\mathbf{Q_{(x)}} = \mathbf{G}^{-1}(\mathbf{G}^{-1})^*$$

where $(\mathbf{G}^{-1})^*$ is transposed of $\mathbf{G}^{-1}$.

After executing transformation the corrections v_i can be computed from the $\tilde{\mathbf{v}}$ vector of transformed hyper-matrix, using the equation

$$\underset{(n,1)}{\mathbf{v}} = \underset{(n,n)}{\mathbf{P}}^{-1/2} \underset{(n,1)}{\tilde{\mathbf{v}}}$$

In case of practical computation each columns of the hyper-matrix should be stored one by one on hard disk. Since the actual transformation is being performed in the RAM of a computer, the maximum number of equations and unknowns is limited by the RAM size (free place for at least two columns must be provided in the RAM at the same time). Matrix-orthogonalization method gives a good possibility to solve large equation systems in general RAM size beside high numerical stability, Völgyesi (2001).

To decrease the effect of relatively large errors in the adjustment their weight should be decreased, but before the adjustment the errors are not known. This contradiction can be solved by an iteration (*Danish method*): in the first step (*j=1*) all observed data has equal weight (*p=1*) in the further steps the weight will be:

$$p_{ij} = \frac{1}{1 + a_k\, v_{j-1}^2}$$

where j the actual iteration step. The a_k coefficient is correct if p=0.25 for the erroneous measurement, Soha (1986). The threshold of errors can be taken as the function of the errors of unit weight, then:

$$a_k = \frac{3}{v_k^2}$$

where

$$v_k = 3\mu_0 \qquad \text{if} \qquad v_{max} > 3\mu_0$$

$$v_k = 2\mu_0 \qquad \text{if} \qquad 2\mu_0 < v_{max} < 3\mu_0$$

$$v_k = \mu_0 \qquad \text{if} \qquad \mu_0 < v_{max} < 2\mu_0$$

The erroneous measurements will get less weight by each subsequent iteration step. The iteration should be continued until the error of unit weight is decreased in a considerable way. In the adjustment of MGH–2000 two steps of iteration proved to be sufficient. The data set consisted of all the measurements of MGH–2000, absolute points near to the border in the neighboring countries and connecting ties across the borders, altogether 5544 observed gravity differences and their corrections, 450 unknowns (point values and scale factors of gravimeters) 20 absolute points, 436 gravimetric points including 8 Austrian and 42 Slovakian ones.

7 Conclusions

Fig.2 represents the histogram of the corrections. It can be seen that 97 % of them are less than 45 μGal ($3M_0$). The weight of measurement having higher corrections is so small that they hardly influence the adjusted values. The RMS of the adjusted values is ±6–10 μGal. The RMS of unit weight of the adjusted network is ± 14 μGal.

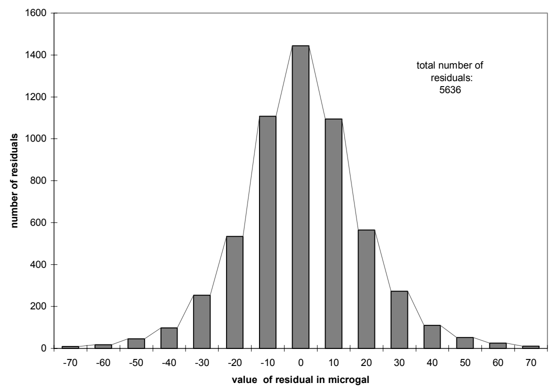

Fig. 2 Histogram of residuals

To check the quality of the adjustments and to compare the networks of the neighboring countries three comparisons were carried out.

In the first comparison we compared the adjusted values of MGH-2000 to the Slovakian results on those points, which were included in the adjustment of both networks. The maximum deviation we have obtained on identical points was 20 μGal. The results can be regarded as excellent taken into consideration the differences of the two networks (different gravimeters, database, adjustment).

In the second comparison we compared the Austrian and Hungarian gravity datum. Based on 8 common points the Hungarian datum proved to be higher than the Austrian by 18 μGal.

In the third comparison we compared the gravity values obtained for common points in the adjustment of UEGN '94 and MGH-2000. We could do this because five Hungarian points which were part of the Austrian-Hungarian interconnecting measurements in 1992-93 were already included in the adjustment of UEGN '94, Boedecker et al. (1993). Based on the five points the Hungarian datum is higher by 17 μGal than the international one (see *Tab. 2*).

Table 2. Comparison of UEGN' 94 and MGH–2000 networks based on identical Hungarian points

Number and name of points		Gravity value [μGal]		Difference [μGal]
UEGN' 94	MGH–2000	UEGN' 94	MGH–2000	
1835 FRTOD	4111 Fertőd	980824222 ± 8,0	980824234 ± 4,6	12
1836 HGYEHAL	4122 Hegyeshalom	980844449 ±12,0	980844470 ± 6,0	21
1837 KESZG	85 Kőszeg	980784705 ±15,0	980784713 ± 5,0	8
1838 SPRO	4105 Sopron	980808350 ±14,0	980808382 ± 5,2	32
1839 VELCJ	4112 Völcsej	980802189 ±14,0	980802203 ± 3,9	14

Acknowledgement

The authors want to express their gratitude to National Committee for Technological Development (OMFB) and U.S.-Hungarian Science and Technology Joint Fund (MAKA) which organizations supported the measurements of MGH-2000 (MEC-94-0508 and JF-369 contracts). Their financial assistance rendered the completion of the project. Special gratitude is due to NIMA for their support of ELGI. We are indebted to *D. Ruess* for the absolute measurements carried out by JILAG-6 gravimeter.

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Csapó G, Völgyesi L (2002): Hungary's new gravity base network (MGH-2000) and it's connection to the European Unified Gravity Net. Vistas for Geodesy in the New Millenium IAG Symposia (Editors: J.Ádám, K.P.Schwarz). Springer Verlag Berlin, Heidelberg, New York, Vol.125, pp. 72-77.

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