# GEODYNAMIC INTERPRETATION OF REPEATED GRAVITY OBSERVATIONS

by

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**Zusammenfassung**

Die Intensität eines Schwerefeldes ist eine Funktion des Ortes. Ist die Verteilung der das Schwerefeld hervorgerufenen Massen zeitlichen Änderungen untergeworfen kann die beobachtete Schwereänderung einer Sration auf der Oberfläche der Masse als Funktion der Änderung des Schwerefeldes und der verlagerung des Beabachtungsortes ausgedrückt werden. Wiederholte geodätische Beobachtungen liefern relative Bewegungen der Beobachtungs stationen in Bezug auf das Schwerefeld der Erde bzw. auf das Geozentrum (den Massenmittelpunkt der Erde). Bis die Bestimmung der wahren Bewegungen des Beobachtungsortes auf der Erdoberfläche nicht möglich wird, bilden die Ergebnisse von wiederholten Schwerebeobachtungen keine ausreichende Informationen um irgendeine Aussage in der Frage der zeitlichen Änderung des Erdschwerefeldes machen zu können.

With the development of our knowledge in geodynamics, the earlier static aspect in connection with the Earth, a part of which, among others, is the assumption being Earth's gravity field constant in time, can be regarded less and less acceptable. Nowadays, on the basis of the modern dynamic view increasingly more people are dealing with the possibility of the non tidal variation of gravity that is the long period and the secular change of Earth's gravity field. These changes are caused by more or less known processes as e.g. the surface tectonic motions or the extensive mass rearrangements assumed in the deeper parts of the Earth. Lately, as a consequence of the recent development of instruments and methods, observations of gravity of special accuracy can be carried out, the accuracy of which surpasses the assumed order of this type of changes of the gravity field.

On the basis of the present practice, from the repeated observations of gravity performed on the physical surface of the Earth authors have concluded to the stability resp. variations of the Earth's gravity field. At the same time other examples show that the observed changes of heights are connected with gravity changes observed just there.

Thus the question arises whether in any Earth surface point the time change of the gravity field or the displacement of the Earth surface for any reason can be calculated only from the change of gravity. To answer the question the change of gravity in time in arbitrary Earth surface point has to be given physical contents – i.e. it has to be interpreted precisely.

The change of gravity for any reason on the surface of the Earth can be given by the differential formula

$$dg = \frac{\partial g}{\partial W} dW \tag{1}$$

However this relation can easily mislead as, at first sight it shows that from the change of gravity $g$ the variation of the potential $W$ of the gravity field can be calculated. If the physical contents of $dW$ is examined then we can be convinced easily that actually it is not possible.

The $dW$ in relation (1) represents the total change of potential in any point of the physical surface of the Earth. This change of the potential can be traced back to two independent reasons. One of the possible reasons is the displacement and rearrangement of the inner masses generating Earth's gravity field (e. g. the displacement of the solid core of greater density of the Earth in the fluidal outer core of smaller density). As a result of it in given Earth surface point both the gravity itself and its potential will change, termed usually as the non tidal variation of gravity field. Another cause of the change of potential can be that as a consequence of different surface geological processes (e. g. rock compaction, etc.) the station itself shifts in space, and by this it gets to a location with an other potential.

Let's examine, first, in case of inner mass rearrangement the change of potential in any arbitrary Earth surface point assumed motionless.

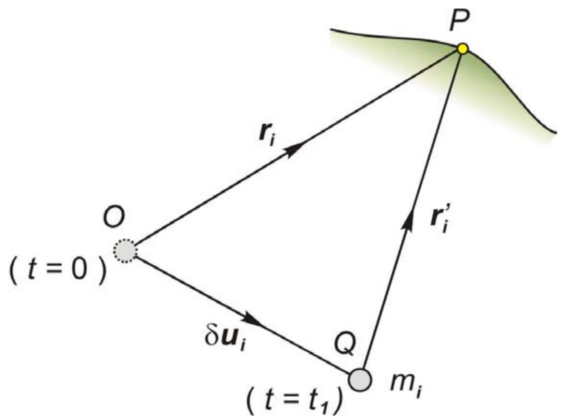


*Fig. 1*

In epoch $t = 0$ the potential in the surface point generated by the mass point $m_i$ inside of the Earth in point $O$ is

$$W_i = k \frac{m_i}{r_i}$$

where $r_i$ is the distance between points $O$ and $P$. Now if for epoch $t = t_1$ the mass point $m_i$ moves to point $Q$ from its original position in the way seen in Fig. 1, the potential accordingly will be:

$$W_i' = k\frac{m_i}{|\vec{r}_i - \delta\vec{u}_i|} \quad .$$

Then in point $P$ assumed to be in unchanged position the variation $\delta W_i^{(u)}$ of the potential appearing because of the displacement $\delta\vec{u}_i$ of mass point $m_i$ is

$$\delta W_i^{(u)} = W_i' - W_i = k\frac{m_i}{r_i}\left(\frac{r_i}{|\vec{r}_i - \delta\vec{u}_i|} - 1\right) \quad . \tag{2}$$

If not only one mass point of Earth's core but a system of $n$ points does change its location then the change of potential in the surface point $P$ will be

$$\delta W^{(u)} = \sum_{i=1}^{n} \delta W_i^{(u)} = k\sum_{i=1}^{n}\frac{m_i}{r_i}\left(\frac{r_i}{|\vec{r}_i - \delta\vec{u}_i|} - 1\right) \quad . \tag{3}$$

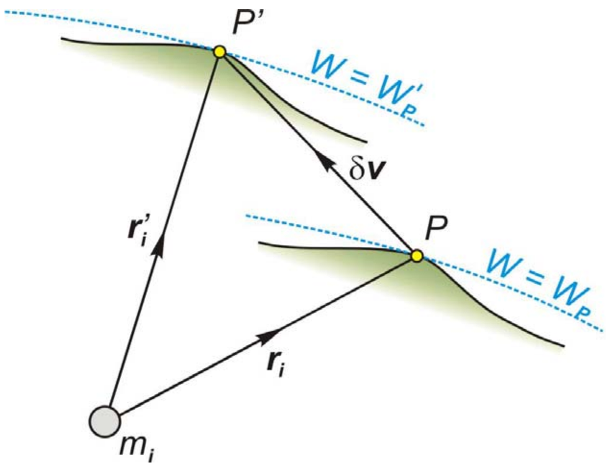


*Fig. 2*

Let's suppose that in the Earth's core there is no kind of mass rearrangement and let's examine now the case when the potential will be changed only because of the displacement of the Earth's surface. In this case, if the observation station moves in the gravity field of mass point $m_i$ from point $P$ with the potential $W_P$ to point $P'$ with the potential $W_{P'}$ then the potential change corresponding to the displacement $\delta\vec{v}$ (Fig. 2) can be expressed by

$$\delta W_i^{(v)} = k\frac{m_i}{|\vec{r}_i + \delta\vec{v}|} - k\frac{m_i}{r_i} \quad .$$

This will be in the case of $r_i >> v$ and taking only the vertical component $\delta r$ of the surface displacement of general direction $\delta\vec{v}$ into consideration

$$\delta W_i^{(r)} = -k \frac{m_i}{r_i^2} \delta r \quad ,$$

and finally for the Earth

$$\delta W^{(r)} = -g\, \delta r \quad . \tag{4}$$

Further on let's examine the general case when the potential change $dW$ in (1) takes place as a consequence of the common existence of the previous two reasons. For this case the absolute value of the change of gravity can be calculated by using Eq. (1) in the form

$$\delta g = \frac{\partial g}{\partial W}\left(\delta W^{(u)} + \partial W^{(r)}\right) \quad ,$$

or regarding (4):

$$\delta g = \frac{\partial g}{\partial W} \delta W^{(u)} - \frac{\partial g}{\partial W} g\, \delta r \quad ,$$

being $\delta W^{(r)}$ the change of potential caused by the vertical displacement $dr$ of the station and $\delta W^{(u)}$ the potential change in consequence of the physical reasons (rearrangement of masses) taking place in the Earth's interior. However the partial derivative $\partial g / \partial W$ can be written in another way:

$$\frac{\partial g}{\partial W} = \frac{\partial g}{\partial r}\frac{\partial r}{\partial W} = -\frac{1}{g}\frac{\partial g}{\partial r}$$

where $\partial g / \partial r$ is the vertical gradient of gravity. Considering this and neglecting the superscript *(u)*

$$\delta g = \frac{\partial g}{\partial W} \delta W - \frac{\partial g}{\partial r} \delta r \quad . \tag{5}$$

Among the quantities in (5) the variation of gravity $\delta g$ on the left side of the expression can be determined by observations and from it we usually want to deduce the change of $\delta W$ – the non tidal variation of the gravity field.

Our repeated observations of gravity can lead to two types of result: we can find either that within the observed period $\delta g = 0$ , i.e. gravity has not changed; or repeated observations result in other values thus $\delta g \neq 0$.

First let's examine the case when $\delta g = 0$ will be observed. On the basis of (5) this can occur in two cases:

One case is when $\delta W = \delta r = 0$ ; i.e. it has actually nothing happened − thus neither the gravity field of the Earth has been changed ($\delta W = 0$) nor the station has moved ($\delta r = 0$) . Knowing the dynamism of the Earth this possibility is hardly feasible.

The other possibility of observed $\delta g = 0$ is if

$$\frac{\partial g}{\partial W}\delta W = -\frac{\partial g}{\partial r}\delta r \quad , \tag{6}$$

thus the change of the gravity field will be compensated by a deformation of the Earth's surface corresponding with it. This could be experienced in the case of a model Earth, where the lithosphere reacts as fluidity to the long period secular changes of gravity field. Or reversed we can say: if 1ithosphere behaved as fluidity we could observe only $\delta g = 0$ variations on the surface, but according to (5) we must not conclude form them that in the meantime the gravity field has not been changed. So it can be stated that under the assumption of a lithosphere reacting as fluidity it is in principle impossible to determine the time variation of Earth's gravity field only on the basis of repeated observations of gravity carried out on the Earth's surface.

Another possible result of the repeated observations of gravity can be that within the observed period gravity has been changed, thus $\delta g \neq 0$ . If this will be derived from the observations the lithosphere cannot be considered to behave as fluidity because in the case of $\delta g \neq 0$ (6) does not exist.

$\delta g \neq 0$ can occur in the following three cases. According to a first possibility $\delta g \neq 0$ can arise so that $\delta r = 0$ consequently $\delta W \neq 0$. In this case the change of potential is a consequence of internal mass rearrangements; $\delta r = 0$ means that lithosphere is entirely rigid, thus it is not able for any kind of deformation. As an example let's imagine a model Earth having a rigid crust, a fluid external core and an eccentric solid inner core. If the inner core of greater density moves in the fluid material of the external core of smaller density, the change of gravity field can be experienced at the rigid Earth surface. In this case − excluding the possibility of surface displacements − on the basis of the repeated gravity observations $\delta W$ can be calculated − that is the time variation of the gravity field can be determined. After having considered the further possibilities we shall see that the variation of the gravity field can be determined by gravity observations at the Earth's surface only in this exceptional case.

According to an other possibility $\delta g \neq 0$ can be imagined if $\delta W = 0$ and $\delta r \neq 0$. So if we assume that $\delta W = 0$ − i.e. a constant gravity field − then the experienced change of gravity can be caused only by the displacement of the Earth surface point. Such surface displacements can take place because of local geological (tectonic) reasons, as a consequence of which the observed point gets at a new location of another potential value in the space. In this case, the change of Earth's gravity field is not questionable, as its possibility has been excluded. So it can be stated that the vertical displacement of any point, at the Earth's surface can be determined on the basis of repeated gravity observations using Eq. (5) only under the condition if the real change of the gravity field has been excluded.

At last a third case is the general case, when $\delta g \neq 0$ if $\delta W \neq 0$ and $\delta r \neq 0$ ; i.e. the observed change of surface gravity appears as the resultant of the two effects. In this variant the real change of the field can be determined only in two special cases on one hand if $\delta r$ could be observed in some way; on the other hand if $\delta r$ could be expressed as any known function of $\delta W$ . First let's deal with this second possibility. If the displacement of the Earth surface $\delta r$ were any function of the change of gravity field then it could be expressed mathematically in the form

$$\delta r = f(\delta W) \quad . \tag{7}$$

Substituting it into (5) it would be:

$$\delta g = \frac{\partial g}{\partial W}\delta W - \frac{\partial g}{\partial r} f(\delta W) \quad . \tag{8}$$

On the basis of relation (8) in the knowledge of the function (7) the change of gravity field could be determined unequivocally. However the determination of the function (7) is not a simple task, it can be done only approximately in cases of adequate Earth models. Such a function can be given e.g. on the basis of Love's theory of elasticity:

$$\delta r = h \frac{\delta W}{D g} \tag{9}$$

where $D = 1 - h + k$ is the ratio of the potential change of the real elastic and a totally rigid Earth mass, $h$ and $k$ are Love's numbers characterizing the elasticity of the Earth mass. The figures of $h$ and $k$ are different at different Earth models, e.g. $h = 1$ in case of a fluid lithosphere, $h = 0$ in case of a lithosphere assumed to be entirely rigid and $0 < h < 1$ in case of the elastic lithosphere.

Thus to give relation (9) numerically is possible only by giving a known Earth model.

In the case of the real Earth a further term must be added to relation (9) because of the local geological-tectonical movements

$$\delta r = f(\delta W) + b$$

being $b$ the unknown surface displacement of the Station.

Finally the real change of the gravity field could be determined in the general case of $\delta W \neq 0$ and $\delta r \neq 0$ even in that case if $\delta r$ could be observed in some way. Then $\delta W$ could be calculated with known $\delta g$ and $\delta r$ through Eq. (5).

But unfortunately $\delta r$ is composed of two parts as it can be seen in Fig. 3:

$$\delta r = \delta N - \delta H \quad ,$$

being $\delta H$ the change of height of the Earth surface point $P$ as compared to the displaced level surface with potential $W' = W_P$ and $\delta N$ the vertical displacement of the level surface – that is unknown and so far we do not know any possibility to determine it.

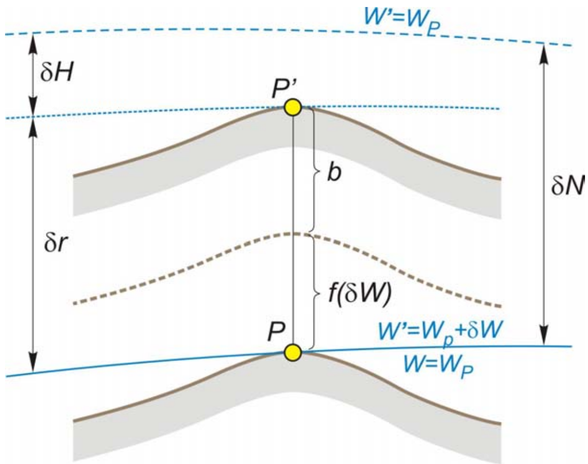


*Fig. 3*

***Summarizing*** the facts mentioned so far we can state that in any Earth surface station only from the observed change of gravity, in the cases of practice, unambiguous conclusion can be drawn neither to the time variation of Earth's gravity field nor to the real vertical displacement of the surface point. If the repeated observations of absolute gravity at the same Earth surface station significantly differ from each other, our only conclusions can be that lithosphere does not behave as fluidity.

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