Some possible physical reason of time variation of Earth's gravity field (a possible proof of time change of gravitational constant)

L. Völgyesi
Department of Geodesy and Surveying, Budapest University of Technology and Economics; Research Group of Physical Geodesy and Geodynamics of the Hungarian Academy of Sciences, H-1521 Budapest, Hungary, Müegyetem rkp. 3.

Abstract. The gravity field of the Earth is a resultant of three different fields, the gravitational, the centrifugal and the tidal one. Time variation of each component causes the time variation of gravity. At present the measurements by gravimeters reach so high accuracy that non tidal variations of gravity need to be taking into consideration. The nGal sensitivity of the most modern superconducting gravimeters may require taking into consideration of some special new gravity effects. Two peculiar effects are discussed here. The supposed time variation of gravitational constant (suggested by the Nobel prise winner physicist Dirac in 1937) may cause the decreasing of gravity by the value of 0.1 μGal/year. The other effect is the supposed expanding Earth (suggested by the Hungarian geophysicist Egyed in 1970) which may cause the decreasing of gravity by the value of 0.2 μGal/year.

Keywords: time variation of gravitational constant, gravity effect of expanding Earth.

Introduction

Around 1583 Galileo observed that the period time of an oscillating pendulum is constant when he compared a lamp's swing in the cathedral of Pisa with his own pulse. It turned out in a short time that the pendulum's position related to the Earth has an effect on the duration of pendulum's oscillation; the pendulum oscillates faster at sites of lower altitude or closer to the poles than in higher mountains and closer to the Equator. With this the possibility to study the Earth's gravity field has opened. With the first mathematical pendulum measurements the value of gravitational acceleration could be determined only with three significant digits, i.e. with an accuracy as low as $10^{-3}\ m/s^2$. Later one managed to achieve the accuracy of mGal with special physical pendulums (1 Gal = 1 cm/s^2 – this name was born just in honor of Galileo). Application of spring gravimeters using the basic principle of astatizing has brought the real breakthrough in gravity field's measurement, the measurement accuracy of $10^{-6} \div 10^{-7}\, m/s^2$ achievable with this already required respect of time changes, i.e. effect of tidal phenomena in gravity field. Development of measuring devices' performance can be followed in Table 1.

Table I. Development in accuracy of gravity measurements

measuring devices	Relative accuracy [m/s^2]	level
simple mathematical pendulum	$10^{-3} \div 10^{-4}$	
special physical pendulums, reversion pendulums	10^{-5}	mGal
earlier quartz gravimeters (Askania, Sharp, Scintrex)	$10^{-6} \div 10^{-7}$	
up-to-date (LCR) gravimeters	$10^{-7} \div 10^{-8}$	
absolute laser gravimeters	10^{-8}	μGal
Superconducting gravimeters	10^{-11}	nGal

In our days the instruments used in measurement of gravity field have reached the accuracy which necessitates intense study of non-tidal time changes in gravity field as well. The *nGal* measurement accuracy level of most up-to-date superconducting gravimeters already requires study of time changes in all imaginable gravity effects and to take them into account. In what follows we review and study several potential physical and geophysical effects respect of which has not been reasonable up to now.

1. Possible reasons for time changes

The force of gravity **F** acting on a body of mass *m* at any point of the Earth (i.e. the body's weight) is:

$$\mathbf{F} = \mathbf{F_n} + \mathbf{F_r} + \mathbf{F_t} \tag{1}$$

where $\mathbf{F_n}$ is the Newtonian mass attraction acting on the mass m, $\mathbf{F_r}$ is the centrifugal force of rotation and $\mathbf{F_t}$ is the tidal force of extraterrestrial celestial bodies. The corresponding field strength, i.e. acceleration is:

$$\mathbf{g} = \mathbf{g_n} + \mathbf{g_r} + \mathbf{g_t} \tag{2}$$

where

$$|\mathbf{g_n}| = G \int\limits_{r'}\int\limits_{\psi'}\int\limits_{\lambda'} \frac{\vartheta(r',\psi',\lambda')\, r'^2 \sin\psi'}{(r-r')^2} dr'\, d\psi'\, d\lambda' \tag{3}$$

$$|\mathbf{g_r}| = \omega^2 r^2 \cos^2\psi \tag{4}$$

$$\begin{aligned} |\mathbf{g_t}| &= GM_{☾} \frac{r}{r_{☾}^{\,3}}\left(3\cos^2\zeta_{☾} - 1\right) \\ &\quad + GM_{\odot} \frac{r}{r_{\odot}^{\,3}}\left(3\cos^2\zeta_{\odot} - 1\right) + \ldots \end{aligned} \tag{5}$$

Using the notations in *Fig. 1* (Völgyesi 1999, 2005) in an approximate calculation of tidal field only effect of the Moon (☾) and the Sun (⊙) was taken into account.

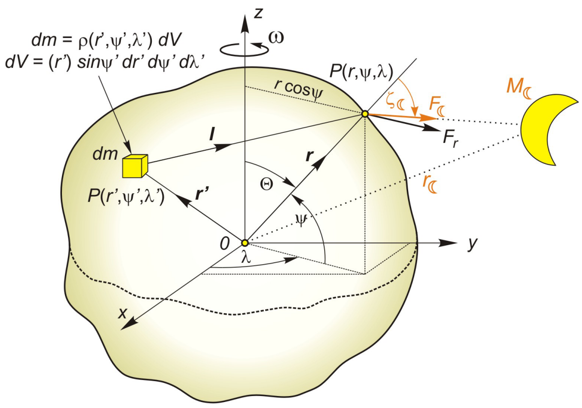

Fig. 1 Notations for calculation of the components of the Earth's gravity force

Considering the effects of mass attraction, of rotational centrifugal and tidal forces based on relationships (3), (4) and (5) it can be determined which are the quantities whose time change can result in time change of gravity field:

$$\begin{aligned} g(t) &= G(t) \int\limits_{r'(t)}\int\limits_{\psi'(t)}\int\limits_{\lambda'(t)} \\ &\quad \frac{\vartheta(r',\psi',\lambda',t)\,[r'(t)]^2 \sin[\psi'(t)]}{[r(t)-r'(t)]^2} dr'\, d\psi'\, d\lambda' \\ &\quad + [\omega(t)]^2 [r(t)]^2 \cos^2\psi(t) \\ &\quad + G(t)M_{☾}(t)\frac{r(t)}{[r_{☾}(t)]^3}\left\{3\cos^2[\zeta_{☾}(t)] - 1\right\} \\ &\quad + G(t)M_{\odot}(t)\frac{r(t)}{[r_{\odot}(t)]^3}\left\{3\cos^2[\zeta_{\odot}(t)] - 1\right\} \end{aligned} \tag{6}$$

In relationship (6) those terms are marked which might play a role in gravity field's time change due to any kind of reason, then based on this we summarized the possible reasons of time change in Table II.

Table II. Possible reasons for time changes of gravity field

$G(t), r(t),$ $M_{☾}(t), r_{☾}(t), \zeta_{☾}(t),$ $M_{\odot}(t), r_{\odot}(t), \zeta_{\odot}(t), \ldots$	Tide
$r'(t)$	mass realignment
$r(t)$	surface movements
$\vartheta(r',\psi',\lambda',t)$	change in density
$\psi(t), \lambda(t)$	polar motion
$\omega(t)$	decreasing rotational angular velocity
$G(t)$	time change in gravitational constant
$r(t)$	expanding Earth

The tenth of mGal magnitude tidal effect shown in the first line of Table II is taken into account in all g measurements (e.g. Wenzel, 1996). Due to the realignment of masses local, regional and global changes in gravity field can develop according to the volume, density distribution and motion velocity of masses taking part in processes that can be secular, short period and random variations. The atmospheric, meteorological changes, the precipitation water, movement of surface and underground water, the eustatic changes, the geological, tectonic processes, the internal mass realignment in Earth and human activity equally can cause significant time changes in gravity field (Völgyesi, 2005). The magnitude of these changes can vary in a very wide scale; in some cases they can reach even the value of several mGals (Tóth-Völgyesi-Cerovsky, 2004). Vertical movements (e.g. in Scandinavia), or the changes in density (e.g. rock compaction, water and oil exploitation) can cause significant changes. As a consequence of pole wandering an almost periodical rotation axis shift of annual 10-20 m magnitude can be observed. Accordingly, the centrifugal acceleration shows a quasi-periodical annual variation of 5 μGal magnitude around the latitude of 45^o, this value fits in well with the measurements (Amalvict-Debeglia-Hinderer, 2003).

Generally, all these variations are commonly taken into account in gravity acceleration measurements, these are not unknown effects. Development of measuring instruments and observation technique might necessitate consideration of other effects, independent of the previous ones as well. In what follows we discuss the gravity effect of the time change in gravitational constant assumed by Dirac (1937) and that of Egyed's theory of the expanding Earth (Egyed, 1970) that is also in connection with it. In addition, rate of change in gravity acceleration due to the Earth rotation's slowing angular velocity will also be briefly checked.

2. Time change of gravitational constant

Constancy of the *G* gravitational constant in the equation describing the Newtonian mass attraction within the measurement accuracy applies only to the minute period of up to now available human observation. Today it is already known that laws of classical mechanics can be considered approximately valid only in the scale of human life, they are not suitable to describe very large size and high velocity phenomena. Similarly, it turned out that e.g. the classical electromagnetic theory cannot be applied in the very small, world of atomic size. We have also no reason to assume that laws of classical physics operate in every respect in the same way during the infinitely long history of Universe as in the very short period observed by us.

From the nearly identical values of dimensionless numbers deduced from some universal constants *P.A.M. Dirac,* Nobel-prize winner physicist came to the conclusion that the value of gravitational constant is inversely proportional to the age of Universe (or a part of the Universe); value of gravitational constant decreases with time (Dirac, 1937). Later Gilbert deduced this idea from the general theory of relativity using certain assumptions (Gilbert, 1956). Today time change in gravitational constant is not a proven theory yet, although it is an ever more accepted hypothesis among theoretical and astrophysicists.

Decrease in gravitational constant's value can be brought into close connection with the phenomenon well known from astronomical observations, the so called *Hubble-effect*. Its essence is that according to the so called redshifts that can be experienced in spectroscopy shows that the further is a galaxy from us the faster it diverges. Ratio of the moving off velocity and distance is constant; its value is the Hubble-constant *H*. In relativistic dynamic models compatible with astronomic observations the magnitude of time elapsed from the beginning of expansion is $T = 1/H$, i.e. about 10 billion years, so the likely value of Hubble-constant is: $H \approx 3 \cdot 10^{-18} s^{-1}$. Reduction of gravitational constant is very slow, according to Nielsen's (2003) investigations

$$\frac{1}{G}\frac{\partial G}{\partial T} = -\frac{1}{3}\frac{1}{T} \ , \qquad (7)$$

where *T* is the present age of Universe.

Let us try to guess the magnitude of incidental change in Earth's gravity field caused by time change in *G*. In accordance with Newton's II. law and also Newton's gravitational law

$$F = mg = G\frac{mM}{r^2} \quad \rightarrow \quad g = G\frac{M}{r^2} \ ,$$

Its time change is

$$\frac{\partial g}{\partial t} = \frac{M}{r^2}\frac{\partial G}{\partial t} + GM\frac{\partial}{\partial t}\left(\frac{1}{r^2}\right)\frac{\partial r}{\partial t} \ .$$

After rearrangement

$$\frac{1}{g}\frac{\partial g}{\partial T} = \frac{1}{G}\frac{\partial G}{\partial T} - \frac{2}{r}\frac{\partial r}{\partial T} \ . \qquad (8)$$

Because according to *Nielsen (2003)*

$$\frac{1}{r}\frac{\partial r}{\partial T} = -\frac{1}{G}\frac{\partial G}{\partial T} \ , \qquad (9)$$

therefore taking into account (7), (8) and (9)

$$\frac{1}{g}\frac{\partial g}{\partial T} = -\frac{1}{T} \ .$$

Based on this the *g* values shown in *Table III* can be calculated for different moments of Earth's history.

Table III Change in *g* value as a consequence of gravitational constant's reduction going backward in time, after Nielsen (2003)

T [10^9 years]	0	0.5	1	2	3	4
g [m/s^2]	9.8	10.3	10.8	12.1	13.7	15.9

Based on the data in *Table III* it can be laid down that the change in *g* is not steady, the decline is ever slower. Similarly, based on the data in *Table*

III it can be seen that the value of change in the value of g is about 1 m/s^2 in the last 1 billion years, this corresponds to an *annual diminution of 0.1 μGal*. Its detection is theoretically feasible with the most up-to-date superconducting gravimeters.

It is imaginable that the assumed extremely small annual diminution in the gravitational constant known at present with an accuracy of only few decimals can be detected in this way in a short time, - provided that our theoretical considerations are correct, further effect of change in gravitational constant can be separated from several other gravitational effects.

3. Effect of the expanding Earth

The hypothesis of the Earth's expansion is fundamentally attached to the name of Egyed (1970), and since then the idea has stood in the focus of several geophysicists' interest (e.g. Carey, 1976; Scalera-Jacob, 2003). Nowadays Egyed's idea has not been proved yet and an accurate justification dealing with all minute details is a difficult task of the future.

To explain the phenomenon it is necessary to know the astrophysical Earth model. According to this model the material differentiation is ever weaker moving toward the Earth centre, because there is no adequate reason for development of belt-like material separation. Consequently, the astrophysical model explains the presence of the mantle-core boundary and of inner core by physical phase transition. Study of the stars' internal structure provided the physical basis for the model. There are star types (e.g. the white dwarf stars) in which the material is not of molecular structure due to the high density but it is in plasma state. To reach this the material's atoms should get close to each other due to any kind of reason, and thus the outer electron shells of nearby atoms step into interaction with each other complying with Pauli's principle of quantum mechanics (Jeffreys, 1976).

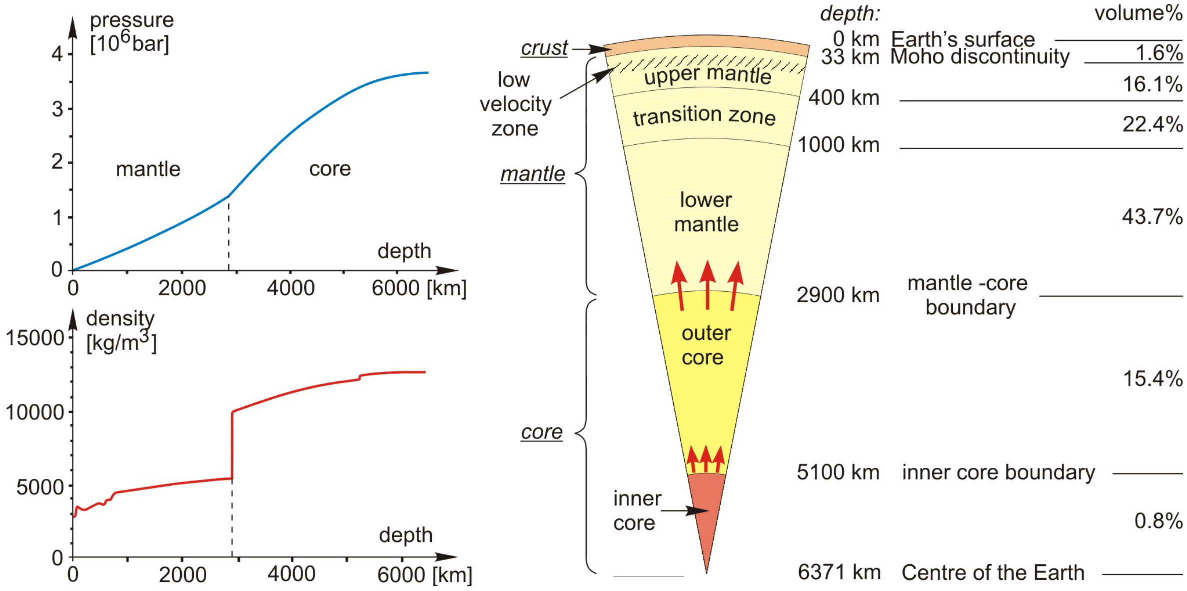

Fig. 2 Structure of the Earth and its major physical parameters

According to the astrophysical Earth model the atoms in the Earth's interior take up the pressure by means of their electron shell structure. The high pressure in the mantle forces the particles into a latticework and thus each atom's position related to its vicinity is determined; material is in the so called *normal phase* state. When any kind of force, e.g. earthquake wave dislocates the particles from their positions, the forces generated by the forced latticework force them in the form of shear force back into their original positions. As a consequence, shear waves propagate everywhere in the whole mantle. The discontinuity between the mantle and core is a horizon of critical pressure value, where material gets into a special plasma state (the so called state of *ultraphase I* of ultra high pressure, and basically only the Coulomb-type electrostatic forces act between the particles. This state of ultraphase I is associated with a decrease in volume, therefore there is a jump-like increase in density at a depth of

2900 km as shown in *Fig. 2*, and similarly, this plasma state is the reason for termination of shear stresses in the outer core. With further increase in pressure, however, moving toward the Earth centre particles get so close to each other that in spite of the plasma state they are fain to get into a lattice-like work, into the so called state of *ultraphase II* to take up the pressure. A consequence of such a lattice-like work is, however, a jump-like increase again in density and re-appearance of shear forces at a depth of about 5000 km, at the boundary of the inner core.

Let us investigate what happens in the Earth's interior in the case of gravitational constant's assumed diminution. Then, the value of *g* decreases in the Earth's interior and therefore assuming a hydrostatic pressure the value of pressure also decreases. Due to the decrease in pressure material retransforms at a depth of 5000 km, from the instable ultraphase II of higher intrinsic energy into the ultraphase I, and at a depth of 2900 km. along the mantle-core boundary from the ultraphase I into the normal phase, both phase transitions are associated with a decrease in density and with an increase in volume.

Egyed looked for different evidences for expanding Earth. According to him the upper rock range of granite-like composition developed as a consequence of the differentiation caused by the significant gravity at the beginning of Earth's history, this layer became solid due to the thermal radiation and evolved the present continental crust of very old age. Due to the Earth's increasing volume this primary granite crust rived at several places and development of the young secondary basalt crust of higher density started, its formation continues even today along the oceanic rifts. There is a possibility to estimate the initial Earth radius or the rate of radius increase from the ratio of the area of the continental granite crust and the present entire Earth surface in case of certain condition's fulfillment. According to Egyed's calculations the Earth's radius might be 3500-4000 km at the time of granite crust's formation, about 4 billion years ago; therefore the increase in Earth's radius is about 2500 km in 4 billion years, i.e. nearly 0.6 mm a year. Egyed obtained similar values from other geological studies (Egyed, 1970).

Calculating with the normal value of vertical gradient the annual 0.6 mm radius increase results in an annual decrease of almost 0.2 μGal magnitude in gravity acceleration,. This means that the increase in Earth's radius may be detectable with the present absolute laser gravimeters in the course on one-two decade long repeat measurements applying proper measurement logic, if we are able to separate it from other gravity effects (Biró, Völgyesi 1981; Biró 1983).

4. Slowing angular velocity of Earth's rotation

Since the angular velocity of Earth's rotation around its axis is not steady the gravity field also changes due to the change in centrifugal force. This change in gravity field is a function of the separation from the axis of rotation, therefore it is the highest along the Equator on the Earth's surface, moving to the north and south of this it decreases, and it cannot be observed at the poles.

The rate of angular velocity's time change can be described by the

$$\dot{\omega} = \frac{d\omega}{dt} = \frac{d^2\varepsilon}{dt^2}$$

Angular acceleration; where ω is the rotation's angular velocity, and ε is the angle of turn.

The angular velocity of Earth's rotation may have secular (paleo-secular), short period and sudden random changes.

Geological evidences could support the secular slowing in Earth's rotation velocity and to extend to geological times as well (Zharkov et al 1996). Accordingly, Earth rotated significantly faster than today in the older times. According to the studies the decrease in rotation's angular velocity can be considered steady during the entire huge period and its value in good conformity with astronomical observations is: $\dot{\omega} = -(5.5 \pm 0.5) \cdot 10^{-22} s^{-2}$.

Change in angular velocity of rotation around the axis has a great importance primarily because due to the change in centrifugal force changes in time the gravity field and the shape of field's level surfaces as well, thus the Earth's shape, too. This means a significant change in Earth's oblateness during geological times. Gravity and geometric effects of rotation's decreasing angular velocity can be seen summarized in *Table IV*.

Based on the data in this table it can easily be calculated that the decrease in gravity acceleration due to the slowing rotation velocity is approximately 2 nGal/year, it takes up to the value of 1 μGal in about 500 years in the equatorial region. More suggestive than this is the length reduction of equatorial half major axis *a* related to the length of polar half minor axis *b*. The decrease in *a* is 1 cm in 800 years at the Equator, i.e. 0.0125 mm per year.

Table IV Change in value of *g* and Earth's oblateness due to the decrease in rotation's angular velocity

Moment	Year's length	Rotation's angular velocity	Centrifugal acceleration	Earth's shape
At present:	365 days	$\omega=7.2921151 \cdot 10^{-5}$ s^{-1}	g_f= 0.0338777 m/s^2	*a-b* = 22 *km*
4 10^8 years ago	400 days	$\omega=7.9913571 \cdot 10^{-5}$ s^{-1}	g_f= 0.0406863 m/s^2	*a-b* = 27 *km*

Summary

In on our studies we briefly reviewed those causes which result in gravity field's time changes. The assumed time change of gravitational constant can cause a decrease of approximately 0.1 μGal/year in gravity field. Egyed's idea about the expanding Earth may also be in connection with the time change in gravitational constant, the based on this assumable annual radius increase of 0.6 mm can cause an almost 0.2 μGal/year decrease in gravity acceleration. Increase in gravity acceleration due to the retardation of Earth rotation's angular velocity is nearly 2 nGal/year this totals up to a value of 1 μGal in about 500 years in the equatorial region, thus it's no good dealing with this effect. It can be seen that both the assumed decrease in gravitational constant and assumed expansion of Earth can cause a reduction in gravity acceleration. Their joint value is 1-3 μGal/10 years according to our estimation. This could be simply separated from other effects, because these should appear simultaneously in the whole Earth, at every point. Therefore it is important that it isn't enough to carry out the observations aimed at this at a single point, but at several different sites in the Earth and at the same time (Biró, Völgyesi 1981; Biró 1983). We can say that that gravimetry recently has reached the development level when opportunity offers itself to verify the time change in gravitational constant from measurement series of several decades duration performed at different points in the Earth. If based on the results of repeated *g* measurements carried out in a longer period (several decades) a sufficient decrease in the value of *g* cannot be seen simultaneously in the whole Earth, then the decrease in gravitational constant assumed by Dirac and the idea of expanding Earth should very likely be rejected, or at least we estimated the magnitude of these effects' erroneously.

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Dr. Lajos VÖLGYESI, Department of Geodesy and Surveying, Budapest University of Technology and Economics, H-1521 Budapest, Hungary, Műegyetem rkp. 3.
Web: http://sci.fgt.bme.hu/volgyesi E-mail: volgyesi@eik.bme.hu