DETERMINATION OF VERTICAL GRADIENTS OF GRAVITY BY SERIES EXPANSION BASED INVERSION

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All the elements of the Eötvös tensor can be measured by torsion balance, except the vertical gradient. The knowledge of the real value of the vertical gradient is more and more important in gravimetry and geodesy.

Determination of the 3D gravity potential $W(x,y,z)$ can be produced by inversion reconstruction based on each of the gravity data W_x (= g) measured by gravimeters and gravity gradients W_{zx}, W_{zy}, W_Δ, W_{xy} measured by torsion balance. Besides vertical gradients W_{zz} measured directly by gravimeters have to be used as reference values at some points. First derivatives of the potential W_x, W_y (can be derived from the components of deflection of the vertical) may be useful for the joint inversion too. Determination of the potential function has a great importance because all components of the gravity vector and the elements of the full Eötvös tensor can be derived from it as the first and the second derivatives of this function. The second derivatives of the potential function give the elements of the full Eötvös-tensor including the vertical gradients, and all these elements can be determined not only in the torsion balance stations, but anywhere in the surroundings of these points.

Test computations were performed at the characteristic region of a Hungarian plate area at the south part of the Csepel-island where torsion balance and vertical gradient measurements are available. There were about 30 torsion balance, 21 gravity and 27 vertical gradient measurements in our test area. Only a part of the 27 vertical gradient values was used as initial data for the inversion and the remaining part of these points were used for controlling the computation.

Keywords. 3D inversion, gravity potential, vertical gradients of gravity, torsion balance, Eötvös-tensor

1. Introduction

In the last century approximately 60000 torsion balance measurements were made mainly for geophysical prospecting in Hungary. More than two third part of this measurements is in digital database and available for the immediate applications. In recent years, there was a demand for the geodetic application of these torsion balance measurements (Völgyesi et al, 2005). The curvature gradients W_Δ and W_{xy} can be used for interpolation of the deflection of the vertical and determination of the fine structure of the geoid (Völgyesi, 2001, 2005). The horizontal gradients W_{zx} and W_{zy} and the curvature gradients W_Δ and W_{xy} measured by torsion balance can be used for the determination of the vertical gradients. The first method was developed by Haalck (1950), but his method seemed to be too sensitive to the not linear changing of the horizontal and curvature gradients between the neighbouring torsion balance points (Ultmann, 2012). A new possibility for the determination of the vertical gradients is the application of the inversion reconstruction of the gravity potential based on torsion balance measurements (Dodróka and Völgyesi, 2008a, 2008b). First the main principle of the inversion algorithm is discussed here, and than test computation shows the application of this method.

2. The 3D inversion algorithm

Let us choose the 3D gravity potential $W(x,y,z)$ as an expansion in a series of a known set of basis function $\Psi_1....\Psi_P$:

$$W(x,y,z)=\sum_{i=1}^{N_x}\sum_{j=1}^{N_y}\sum_{k=1}^{N_z}B_l\Psi_i(x)\Psi_j(y)\Psi_k(z) \tag{1}$$

where $l=i+(j-1)*N_x+(k-1)*N_x*N_y$ and B_j are unknown coefficients of the expansion in a series. In our computations Legendre polinomials are applied as the basis functions. The constant term is marked by index 1, so the possibility of i=j=k=1 can be precluded, because the potential is unequivocal apart from an additive constant.

The second derivatives of the potential (1) give the *computed* values of the horizontal gradients W_{zx}, W_{zy} and the curvature data W_Δ, W_{xy}, as

$$^{comp.}W_{zx}=\frac{\partial^2 W}{\partial x\partial z}=\sum_{i=1}^{N_x}\sum_{j=1}^{N_y}\sum_{k=1}^{N_z}B_l\Psi_i'(x)\Psi_j(y)\Psi_k'(z) \tag{2}$$

$$^{comp.}W_{zy}=\frac{\partial^2 W}{\partial y\partial z}=\sum_{i=1}^{N_x}\sum_{j=1}^{N_y}\sum_{k=1}^{N_z}B_l\Psi_i(x)\Psi_j'(y)\Psi_k'(z) \tag{3}$$

$$
{}^{comp.}W_{\Delta} = W_{yy} - W_{xx} = \frac{\partial^2 W}{\partial y^2} - \frac{\partial^2 W}{\partial x^2}
$$

$$
= \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \sum_{k=1}^{N_z} B_l \left\{ \Psi_j^{''}(y) \Psi_i(x) - \Psi_j(y) \Psi_i^{''}(x) \right\} \Psi_k(z) \tag{4}
$$

$$
{}^{comp.}W_{xy} = \frac{\partial^2 W}{\partial x \partial y} = \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \sum_{k=1}^{N_z} B_l \Psi_i^{'}(x) \Psi_j^{'}(y) \Psi_k(z) \tag{5}
$$

where the prime "$'$" denotes differentiation with respect the variable of the basis function. These are the gradients, which can be measured by torsion balance. The vertical gradients can be computed as

$$
{}^{comp.}W_{zz} = \frac{\partial^2 W}{\partial z^2} = \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \sum_{k=1}^{N_z} B_l \Psi_i(x) \Psi_j(y) \Psi_k^{''}(z) \ , \tag{6}
$$

and the first derivatives:

$$
{}^{comp.}W_{z} = \frac{\partial W}{\partial z} = \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \sum_{k=1}^{N_z} B_l \Psi_i(x) \Psi_j(y) \Psi_k^{'}(z) \, , \tag{7}
$$

$$
{}^{comp.}W_{x} = \frac{\partial W}{\partial x} = \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \sum_{k=1}^{N_z} B_l \Psi_i^{'}(x) \Psi_j(y) \Psi_k(z) \, , \tag{8}
$$

$$
{}^{comp.}W_{y} = \frac{\partial W}{\partial y} = \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \sum_{k=1}^{N_z} B_l \Psi_i(x) \Psi_j^{'}(y) \Psi_k(z) \, , \tag{9}
$$

where $W_z = g$ is the gravity, $W_x = -g\xi + U_x$, $W_y = -g\eta + U_y$ can be computed from the components of the deflection of the vertical ξ and η, (U is the normal potential) .

Let's introduce the notations at the q-s measurement point (x_q, y_q, z_q) :

$$
A_{ql} = \Psi_i^{'}(x_q) \Psi_j(y_q) \Psi_k^{'}(z_q) \, , \tag{10}
$$

$$
C_{ql} = \Psi_i(x_q) \Psi_j^{'}(y_q) \Psi_k^{'}(z_q) \, , \tag{11}
$$

$$
D_{ql} = \left\{ \Psi_j^{''}(y_q) \Psi_i(x_q) - \Psi_j(y_q) \Psi_i^{''}(x_q) \right\} \Psi_k(z_q) \, , \tag{12}
$$

$$F_{ql} = \Psi_i^{'}(x_q)\Psi_j^{'}(y_q)\Psi_k(z_q)\,, \tag{13}$$

$$P_{ql} = \Psi_i(x_q)\Psi_j(y_q)\Psi_k^{''}(z_q)\,, \tag{14}$$

$$Q_{ql} = \Psi_i(x_q)\Psi_j(y_q)\Psi_k^{'}(z_q)\,, \tag{15}$$

$$R_{ql} = \Psi_i^{'}(x_q)\Psi_j(y_q)\Psi_k(z_q)\,, \tag{16}$$

$$S_{ql} = \Psi_i(x_q)\Psi_j^{'}(y_q)\Psi_k(z_q)\,, \tag{17}$$

so the computable torsion balance data, vertical gradients and the computable first derivative data at the same point:

$$^{comp.}W_{zx}^{(q)} = \sum_{l=1}^{M} B_l A_{ql}\ , \tag{18}$$

$$^{comp.}W_{zy}^{(q)} = \sum_{l=1}^{M} B_l C_{ql}\ , \tag{19}$$

$$^{comp.}W_{\Delta}^{(q)} = \sum_{l=1}^{M} B_l D_{ql}\ , \tag{20}$$

$$^{comp.}W_{xy}^{(q)} = \sum_{l=1}^{M} B_l F_{ql}\ , \tag{21}$$

$$^{comp.}W_{zz}^{(q)} = \sum_{l=1}^{M} B_l P_{ql}\ , \tag{22}$$

$$^{comp.}W_{z}^{(q)} = \sum_{l=1}^{M} B_l Q_{ql}\ , \tag{23}$$

$$^{comp.}W_{x}^{(q)} = \sum_{l=1}^{M} B_l R_{ql}\ , \tag{24}$$

$$ {}^{comp.}W_y^{(q)} = \sum_{l=1}^{M} B_l S_{ql} \; , \tag{25} $$

where $M = N_x N_y N_z - 1$ is the number of coefficients of the series expansion, and $A_{ql}, C_{ql}, D_{ql}, F_{ql}, P_{ql}, Q_{ql}, R_{ql}, S_{ql}$ are computable and known matrix elements at the q^{th} measurement point. All the values of (10) - (17) can be written to a single coefficient matrix (so called Jacobi matrix)

$$ G_{qj} = \begin{cases} A_{qj} & q \le N_1 \\ C_{qj} & N_1 < q \le N_1 + N_2 \\ \vdots & \\ S_{qj} & \sum_{s=1}^{7} N_s < q \le \sum_{s=1}^{8} N_s \end{cases} . \tag{26} $$

Let us introduce the vector notations for the measured quantities:

$$ \begin{aligned} {}^{meas}\mathbf{d} = \Big\{ & W_{zx}^{(1)},\ldots,W_{zx}^{(N_1)},\, W_{zy}^{(1)},\ldots,W_{zy}^{(N_2)},\, W_{\Delta}^{(1)},\ldots,W_{\Delta}^{(N_3)}, \\ & W_{xy}^{(1)},\ldots,W_{xy}^{(N_4)},\, W_{zz}^{(1)},\ldots,W_{zz}^{(N_5)},\ldots,\ldots,\, W_{y}^{(1)},\ldots,W_{y}^{(N_8)} \Big\} , \end{aligned} \tag{27} $$

and for the computed data:

$$ \begin{aligned} {}^{comp.}\mathbf{d} = \Big\{ & {}^{comp.}W_{zx}^{(1)},\ldots,{}^{comp.}W_{zx}^{(N_1)},\ldots,\; {}^{comp.}W_{zy}^{(1)},\ldots,{}^{comp.}W_{zy}^{(N_2)}\;,\ldots \\ & \ldots,{}^{comp.}W_{zz}^{(1)},\ldots,{}^{comp.}W_{zz}^{(N_5)},\ldots,\ldots,{}^{comp.}W_{y}^{(1)},\ldots,{}^{comp.}W_{y}^{(N_8)} \Big\} \end{aligned} \tag{28} $$

using eqs. (18) - (25). The vector of computed data takes the form

$$ {}^{comp.}\mathbf{d} = \mathbf{GB} \; . \tag{29} $$

The q^{th} element of the discrepancy vector of the measured and the computed data:

$$ e_q^{(1)} = {}^{meas.}W_{zx}^{(q)} - \sum_{l=1}^{M} B_l A_{ql} \; , \tag{30} $$

$$ e_q^{(2)} = {}^{meas.}W_{zy}^{(q)} - \sum_{l=1}^{M} B_l C_{ql} \; , \tag{31} $$

$$ e_q^{(3)} = {}^{meas.}W_{\Delta}^{(q)} - \sum_{l=1}^{M} B_l D_{ql} \; , \tag{32} $$

$$e_q^{(4)} = {}^{meas.}W_{xy}^{(q)} - \sum_{l=1}^{M} B_l F_{ql} \ , \tag{33}$$

$$e_q^{(5)} = {}^{meas.}W_{zz}^{(q)} - \sum_{l=1}^{M} B_l P_{ql} \ . \tag{34}$$

$$e_q^{(6)} = {}^{meas.}W_{z}^{(q)} - \sum_{l=1}^{M} B_l Q_{ql} \ , \tag{35}$$

$$e_q^{(7)} = {}^{meas.}W_{x}^{(q)} - \sum_{l=1}^{M} B_l R_{ql} \ , \tag{36}$$

$$e_q^{(8)} = {}^{meas.}W_{y}^{(q)} - \sum_{l=1}^{M} B_l S_{ql} \ , \tag{37}$$

Let our inverse problem be overdetermined and let the function have to be minimized the norm L_2 of the discrepancy vector:

$$E = \sum_{s=1}^{8} \sum_{q=1}^{N_s} (e_q^{(s)})^2 \ , \tag{38}$$

The discrepancy vector of the measured and the computed data:

$$\mathbf{e} = {}^{meas}\mathbf{d} - \mathbf{GB} \ , \tag{39}$$

and substituting this into (38):

$$E = (\mathbf{e}, \mathbf{e}) = \sum_{q=1}^{N} e_q^2 \tag{40}$$

for the vectorial discussion, where $N = \sum_{s=1}^{8} N_s$, the total number of the measured data.

It is well-known, that the potential field should fulfill the Laplace equation $\Delta W = W_{xx} + W_{yy} + W_{zz} = 0$ at the (free air) measurement points. The computed value of ΔW can be written as

$$
{}^{comp.}\Delta W = \frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2} + \frac{\partial^2 W}{\partial z^2} = \\
= \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \sum_{k=1}^{N_z} B_l \Big\{ \Psi_i''(x) \Psi_j(y) \Psi_k(z) + \\
+ \Psi_i(x) \Psi_j''(y) \Psi_k(z) + \Psi_i(x) \Psi_j(y) \Psi''_k(z) \Big\} \tag{41}
$$

or introducing the notation at the q^{th} measurement pont ($q = 1,2,...,N_9$, here N_9 is the number of the points, where $\Delta W = 0$ is required)

$$
H_{ql} = \Psi_i''(x_q) \Psi_j(y_q) \Psi_k(z_q) \\
+ \Psi_i(x_q) \Psi_j''(y_q) \Psi_k(z_q) + \Psi_i(x_q) \Psi_j(y_q) \Psi''_k(z_q) \tag{42}
$$

the computable value of ΔW can be written as

$$
{}^{comp.}\Delta W^{(q)} = \sum_{l=1}^{M} B_l H_{ql} \ . \tag{43}
$$

In order to take into account these restrictions, we extend the data structures in eqs. (27) and (28) so, that

$$
{}^{meas}\mathbf{d} = \Big\{ W_{zx}^{(1)},...,W_{zx}^{(N_1)},\ W_{zy}^{(1)},...,W_{zy}^{(N_2)},... \\
...,\ W_y^{(1)},...,W_y^{(N_8)},\ 0^{(1)},...,0^{(N_9)} \Big\} \tag{44}
$$

$$
{}^{comp.}\mathbf{d} = \Big\{ {}^{comp.}W_{zx}^{(1)},...,{}^{comp.}W_{zx}^{(N_1)},...,\ {}^{comp.}W_{zy}^{(1)},...,{}^{comp.}W_{zy}^{(N_2)}\ ,... \\
...,{}^{comp.}W_y^{(1)},...,{}^{comp.}W_y^{(N_8)},\ {}^{comp.}\Delta W^{(1)},...,{}^{comp.}\Delta W^{(N_9)} \Big\} \tag{45}
$$

Denoting the extended Jacobi matrix as

$$
G_{qj} = \begin{cases} A_{qj} & q \le N_1 \\ \vdots & \\ S_{qj} & \sum_{s=1}^{7} N_s < q \le \sum_{s=1}^{8} N_s \\ H_{qj} & \sum_{s=1}^{8} N_s < q \le \sum_{s=1}^{9} N_s \end{cases} \tag{46}
$$

the extended discrepancy vector of the measured and the computed data:

$$
{}_{ext.}\mathbf{e} = {}^{meas}_{ext.}\mathbf{d} - \mathbf{GB} \,, \tag{47}
$$

and the new functional to be minimized is

$$E_{ext.} = ({}_{ext.}\mathbf{e}\,,\,{}_{ext.}\mathbf{e}) = \sum_{q=1}^{N} {}_{ext.}e_q^2 \tag{48}$$

where $N = \sum_{s=1}^{9} N_s$.

The solution of this inverse problem is based on the condition system

$$\frac{\partial E_{ext.}}{\partial B_l} = 0\,,\ (l = 1,...,M) \tag{49}$$

resulting in the set of normal equations

$$\mathbf{G}^T\mathbf{G}\mathbf{B} = \mathbf{G}^T\ {}^{meas}_{ext.}\mathbf{d}\,. \tag{50}$$

So this inverse problem is linear, vector **B** of expansion in a series' coefficients can be determined by solving the above set of equations

$$\mathbf{B} = \left(\mathbf{G}^T\mathbf{G}\right)^{-1}\mathbf{G}^T\ {}^{meas}_{ext.}\mathbf{d}\,. \tag{51}$$

So the potential function − apart from an additive constant − can be determined at any points of the region covered by torsion balance measurements using the coefficients of expansion in a series of a known set of basis function.

3. Test computations

For checking of the 3D inversion algorithm test computations were performed at the region of the south part of the Csepel-island where torsion balance and vertical gradient measurements are available. This test area is a characteristic region of a Hungarian plate area. Both topographic conditions and the density of the torsion balance points are similar to the average conditions in Hungary. Earlier torsion balance measurements were made here in 1950 for geophysical prospecting, the average distance between the points was about 1-1.5 km. Since the original point density was not enough for our investigations, new torsion balance measurements were made, completed by vertical gradient measurements. The new measurements were made by the support of the OTKA project K60657 managed by G. Csapó between 2006 and 2010 (Csapó et al, 2009a, 2009b). Location of the 4 earlier torsion balance measurement points are marked by squares, the new 30 torsion balance points marked by circles, and the 21 gravity measurements are marked by crosses on Fig. 1. Torsion balance points marked by light and dark circles were measured in 2007 and in 2008 respectively. The 27 vertical gradient measurement points can be seen on Fig. 2, the structure and the spatial distribution of the values of vertical gradients is illustrated by isolines. The values of the isolines on the Fig. 2 is in [mGal/m] (1 [mGal/m] = 10^{-5} [1/s^2] = 10000 [E] =10000 Eötvös Unit), coordinates are in meters in the Hungarian Unified National Projections (EOV) system.

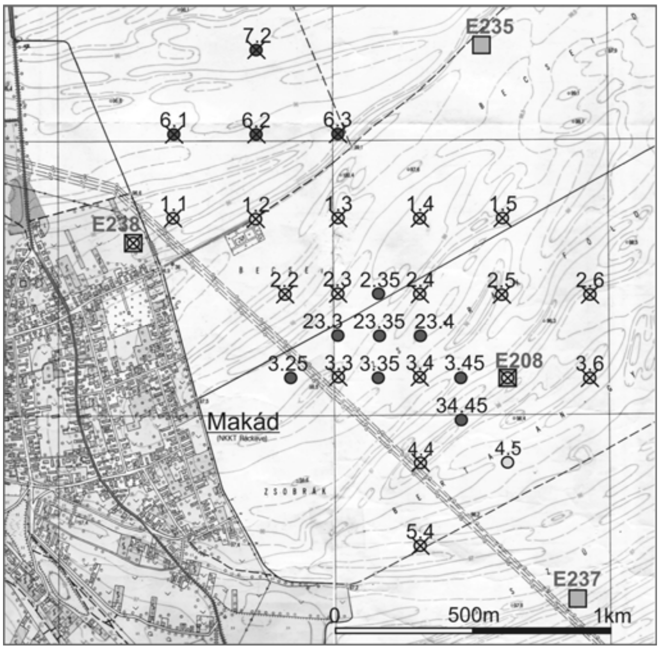

Fig. 1. Torsion balance stations (marked by squares and circles) and gravity measurements (marked by crosses) on the test area.

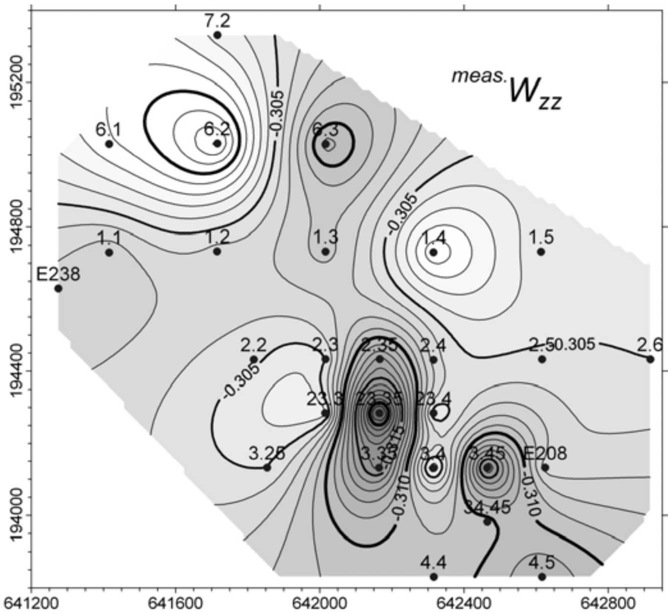

Fig. 2. Vertical gradient measurements on the test area. The W_{zz} values are in mGal/m, coordinates are in meters in the Hungarian EOV system

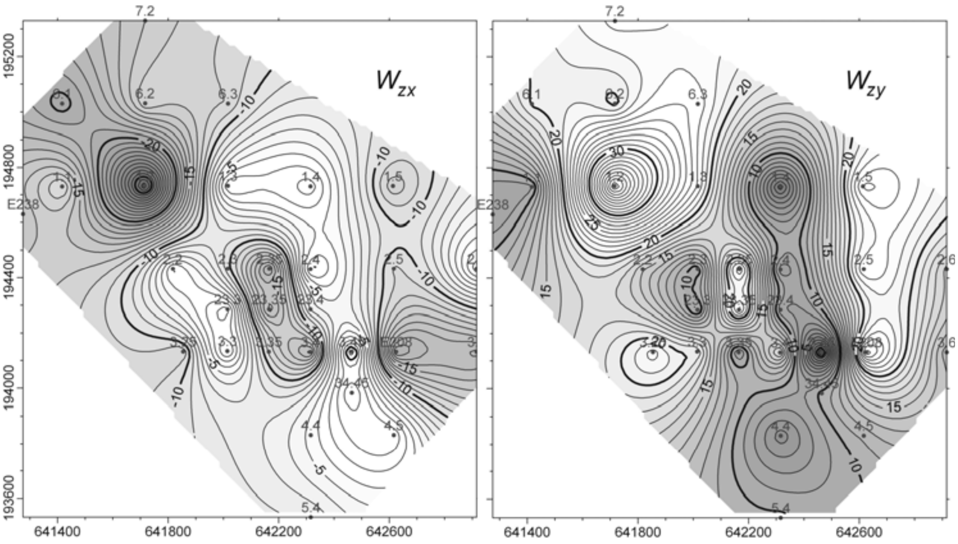

Fig. 3. Isoline map of the horizontal gradients measured by torsion balance.
Isoline values are in [E] (1 E = 1Eötvös Unit = 10^{-9} s^{-2})

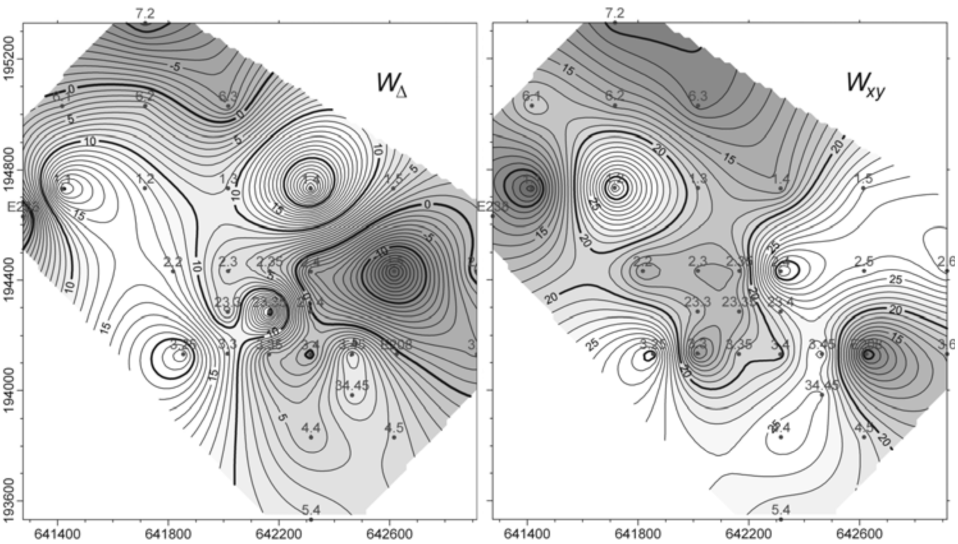

Fig. 4. Isoline map of the curvature gradients measured by torsion balance.
Isoline values are in [E] (1 E = 1Eötvös Unit = 10^{-9} s^{-2})

On the solution of the inversion problem all the coefficients were determined which are necessary for the computation, and all the second derivatives of the potential function were computed by joint inversion for the whole test area. On Figs. 3 and 4 isoline map of horizontal gradients W_{zx} and W_{zy}, and curvature gradients W_{xy} and W_{Δ} can be seen respectively (isoline values are in [E] (1 E = 1Eötvös Unit = 10^{-9} s^{-2}). Comparing the torsion balance measurements to the computed horizontal and curvature gradient data very good agreement can be found. All these gradients from the joint inversion computation are equal within 0.1E accuracy to the measured data and we get the same isoline pictures for the computed gradients as can be seen on Figs. 3 and 4.

In the knowledge of the expansion coefficients, it is also possible to compute the potential field and further first and second derivatives of the potential function by using the expansion formula. So, there is a possibility to determine the vertical gradients W_{zz} from joint inversion in spite of the fact that vertical gradient can not be measured directly by the torsion balance.

For the vertical gradient computation only a part of the measured 27 vertical gradient values was used as initial data for the joint inversion and the remaining part of the points were used for controlling the computation. As it can be seen on Fig. 5 altogether 21 from the whole 27 points marked by dots was chosen as initial data for the inversion computation and the remaining 6 points marked by crosses were used for controlling the results.

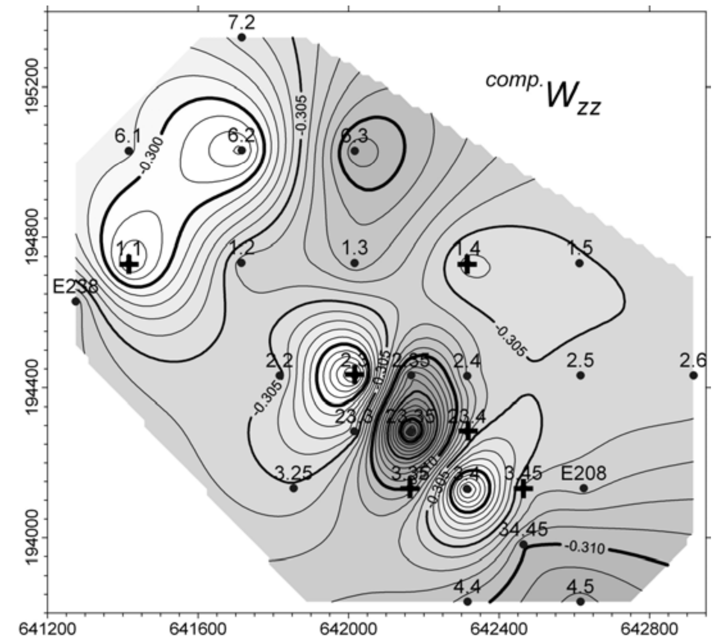

Fig. 5. Computed vertical gradients W_{zz} from the joint inversion, values are in mGal/m

Comparing the measured vertical gradient data to the computed value at the 6 controlling point differences are summarize in Table 1. The root mean square of the differences is ±11.6 μGal/m, which is the order of magnitude of the measurements of the vertical gradient. So this is a strong demostration of the applicability of the inversion reconstruction of the gravity potential for the determination of the vertical gradients based on torsion balance data.

Table 1. Measured and computed data at the controlling points.

Point	measured W_{zz} [mGal/m]	computed W_{zz} [mGal/m]	difference [μGal/m]
1.1	-0.3078	-0.2967	11.2
1.4	-0.3001	-0.3036	-15.6
2.3	-0.3049	-0.2943	16.7
23.4	-0.3042	-0.3062	-0.1
3.35	-0.3164	-0.3097	1.5
3.45	-0.3172	-0.3063	12.3
			RMS =11.6

4. Conclusion

The presented 3D inversion reconstruction method gives possibility for the determination of the gravity potential function by joint inversion using a large number of torsion balance and gravity data completed a few astronomical (deflection of the vertical) and digital terrain model data.

Different important data fields, all the first and the second derivatives of the potential (the full Eövös-tensor, deflections of the vertical, vertical gradients) can be originated from this reconstructed potential function, not only in the torsion balance points but in their surroundings too (at any points of the investigated area).

This method gives a new possibility to transform the torsion balance measurements to different heights and the analytical determination of the geoid surface.

Our investigations haven't yet finished, several questions of detail need to be solved, but the 3D inversion method is working demonstrably well and can be used for the determination of the vertical gradients based on torsion balance measurements completed by different gravity data.

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