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# PHYSICAL BACKGROUNDS OF EARTH'S ROTATION, REVISION OF THE TERMINOLOGY

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Rotation of the Earth is a quite involved process. Deep knowledge of certain areas of physics is indispensable for its understanding and researching. It is necessary to clarify the physics of rigid bodies' rotation because the usage of terms precession and nutation by experts in geosciences are generally not correct, sometimes confused. In physics nutation is the motion of a free gyroscope, and precession is the motion of a heavy one. The torque of an external force causes the precession of a heavy gyroscope, but nutation is a motion of a free gyroscope, if its rotation has not started around its symmetry axis. While the cause of precession of a rotating body always arises from the effect of external mass sources, nutation is exclusively a function of the intrinsic mass distribution of the rotating body. After discussing some important aspects of rotational mechanics (motion of a free and a heavy gyroscope), the Earth's nutation and certain elements of Earth's precession (normal precession, lunisolar precession, planetary precession, disturbing precession) are defined and discussed. A new terminology is proposed, e.g. disturbing precession instead of the widely spread deceptive expression of astronomical nutation.

**Keywords:** Earth's rotation, normal precession, lunisolar precession, planetary precession, disturbing precession, free nutation.

## 1 Heavy and free gyroscopes

Two important types of gyroscopes, the *heavy* and *free* ones, can be seen in Fig. 1. Under the influence of the torque acting on its center of gravity the heavy gyroscope makes *precessional* motion around its symmetry axis in the case of a suitable rotational angular velocity of $\vec{\omega}$, i.e. the rotation axis together with the body wanders round along the surface of a cone with an angular velocity of precession $\vec{\omega}_{pr} << \vec{\omega}$.

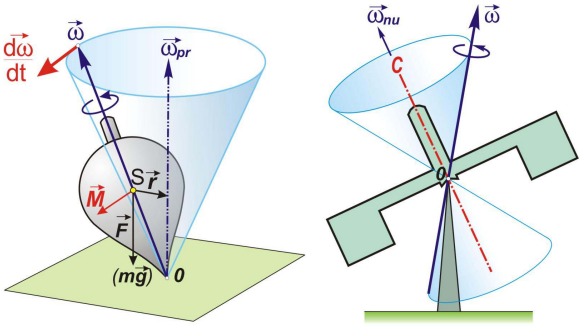


Fig. 1. Motion of a heavy and a free gyroscope

The free gyroscope differs from this in the fact that the torque of external forces respective to its center of gravity is zero (such is e.g. the gyroscope supported at its center of gravity). If the axes of rotation and symmetry of the free gyroscope do not coincide then it makes nutational motion. In this case the body's rotation axis continuously changes its position related to the body, the rotation axis wanders round along the surface of a cone around the body's symmetry axis with an angular velocity of nutation $\vec{\omega}_{nu} << \vec{\omega}$ .

## 2 Precession of heavy gyroscopes

During its motion each rigid body tries to maintain theirs rotation state due to theirs rotation inertness, in other words the angular momentum $\mathbf{N}$ of any closed system is constant according to the law of conservation of angular momentum, therefore its variation in time is:

$$\frac{\mathrm{d}\mathbf{N}}{\mathrm{dt}} = 0\,. \tag{1}$$

If external forces also exert effect on the rotating rigid body, then the change in the angular momentum is equal to the torque $\mathbf{M}$ of the external forces:

$$\frac{d\mathbf{N}}{dt} = \mathbf{M}\,. \tag{2}$$

The torque vector is the cross product of the force $\mathbf{F}$ and the lever arm $\mathbf{r}$ , $\mathbf{M} = [\mathbf{F} \times \mathbf{r}]$ , and according to the relationship known in mechanics the angular momentum is $\mathbf{N} = \mathbf{I}\,\vec{\omega}$ ($\mathbf{I}$ is the inertia moment tensor of the rigid body, and $\vec{\omega}$ is the vector of rotational angular velocity). As $\mathbf{I} = const.$ for a rigid body, therefore $\mathbf{I}$ can be taken out of the differential sign, thus (2) can be written in the form of

$$\mathbf{I}\frac{\overrightarrow{d\omega}}{dt} = [\mathbf{F} \times \mathbf{r}] \tag{3}$$

as well. On the other hand, it can already directly be seen from this, that in the gravity field, under the influence of the external torque the spatial direction of the angular velocity vector $\vec{\omega}$ of rigid, sufficiently rapidly rotating bodies (the so-called heavy gyro-

scopes) continuously changes; the vector $\vec{\omega}$ always moves perpendicularly to the directions of **F** and **r**. In compliance with this the quickly rotating gyroscope of tilt axis that can be seen in the left side of Fig. 1 (e.g. the toy gyroscope, the humming top) does not fall down, but its rotation axis slowly wanders round along the surface of a circular cone of vertical axis, with a constant angular speed of precession $\vec{\omega}_{pr} << \vec{\omega}$. This motion of the gyroscope's rotation axis is the *precession*.

## 3 Motion of symmetric gyroscopes

Let us study the motion of the symmetric gyroscope in that case when both the fixed point $O$ and the center of gravity $S$ are on the symmetry axis, and let denote $s$ the interval $\overline{OS}$. Be the $z$ axis of the $K(x,y,z)$ inertial system fixed in space directed vertically upwards as it can be seen in Fig. 2, on the other hand, directions of the axes of the $K'(x',y',z')$ system fixed to the body and rotating together with it should one after another coincide with the directions of the major moments of inertia $A$, $A$, $C$.

Be the components of the rotational angular velocity vector $\vec{\omega}$ in the system $K'(x',y',z')$: $\omega_{x'}$, $\omega_{y'}$, $\omega_{z'}$ (the body rotates not round its symmetry axis $C$!) and let us characterize the position of system $K'$ in $K$ with the $\vartheta,\psi,\varphi$ Euler-type angles, as it can be seen in Fig. 2. Our task is to describe the motion of the symmetric gyroscope in the system $K$.

To solve the task, let us start from the energy conservation's principle. According to this for the heavy gyroscope the sum of the rotational and the potential energies $mgs\cos\vartheta$ as shown in Fig. 2 is constant:

$$\frac{1}{2}\left(A\omega_{x'}^2 + A\omega_{y'}^2 + C\omega_{z'}^2\right) + mgs\cos\vartheta = const. \tag{4}$$

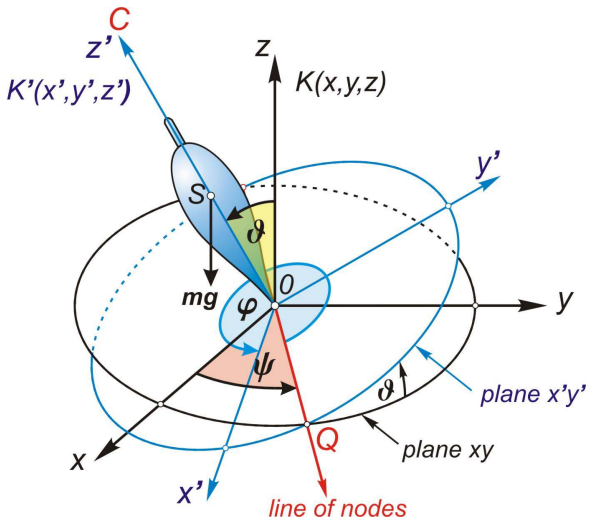


Fig. 2. Coordinates used to describe the motion of the heavy gyroscope

In addition, as it also can be seen in Fig. 2, the torque vector **M** of the force of gravity is perpendicular to both of axes $z$ and $z'$, therefore the components of **M** respective to these two axes are zero:

$$M_z = 0 \tag{5}$$

and

$$M_{z'} = 0\,. \tag{6}$$

From this a further important relationship arises using

$$\left.\begin{aligned} A\frac{d\omega_{x'}}{dt} + (C-B)\,\omega_{y'}\omega_{z'} &= M_{x'} \\ B\frac{d\omega_{y'}}{dt} + (A-C)\,\omega_{x'}\omega_{z'} &= M_{y'} \\ C\frac{d\omega_{z'}}{dt} + (B-A)\,\omega_{x'}\omega_{y'} &= M_{z'} \end{aligned}\right\} \tag{7}$$

the third member of Euler's gyroscopic equations. Taking into account (6), and due to the symmetry equivalence of the moments of inertia, $A = B$ in the plane perpendicular to the rotation axis:

$$C\frac{d\omega_{z'}}{dt} = 0$$

from this, because $C \neq 0$:

$$\omega_{z'} = \omega_{z'0} = \acute{a}ll. \tag{8}$$

At the same time, from equation (5), based on the relationship (2) concerning the conservation of the angular momentum:

$$N_z = const.$$

from this further important conclusions can be done, if this is rewritten, resolving it into components according to the coordinate directions of $K'$ based on Fig. 2:

$$N_z = N_{x'}\cos(x',z) + N_{y'}\cos(y',z) + N_{z'}\cos(z',z) = const.$$

From this taking into account that $N_{x'} = A\omega_{x'}$, $N_{y'} = A\omega_{y'}$ and $N_{z'} = C\omega_{z'}$:

$$N_z = A\omega_{x'}\cos(x',z) + A\omega_{y'}\cos(y',z) + C\omega_{z'}\cos(z',z) = const. \tag{9}$$

Let us express the direction cosines in (9) with the Euler-type angles. Using the relevant relationships of spherical trigonometry, after (BUDÓ, 1964) the individual direction cosines can be expressed with the Euler-type angles as shown summarized in Table 1. (According to the table e.g. $\cos(x',z) = \sin\varphi\sin\vartheta$.)

Writing the appropriate direction cosines in equation (9) from Table 1:

$$N_z = A(\omega_{x'}\sin\varphi\sin\vartheta + \omega_{y'}\cos\varphi\sin\vartheta) + C\omega_{z'}\cos\vartheta = const. \tag{10}$$

Table 1. Direction cosines expressed with the Euler-type angles

| | $x'$ | $y'$ | $z'$ |
|---|---|---|---|
| $x$ | $\cos\varphi\cos\psi$ -$\sin\varphi$ $\sin\psi\cos\vartheta$ | - $\sin\varphi\cos\psi$ -$\cos$ $\varphi\sin\psi\cos\vartheta$ | $\sin$ $\psi$ $\sin$ $\vartheta$ |
| $y$ | $\cos\varphi\sin\psi$ +$\sin\varphi$ $\cos\psi\cos\vartheta$ | - $\sin\varphi\sin\psi$ +$\cos$ $\varphi\cos\psi\cos\vartheta$ | - $\cos$ $\psi$ $\sin$ $\vartheta$ |
| $z$ | $\sin\varphi\sin\vartheta$ | $\cos\varphi\sin\vartheta$ | $\cos$ $\vartheta$ |

Because the components of the angular velocity vector $\vec{\omega}$ in the coordinate system $K'$ rotating together with the body can be expressed with the Euler-type angles

$$\left.\begin{aligned} \omega_{x'} &= \frac{d\psi}{dt}\sin\vartheta\sin\varphi + \frac{d\vartheta}{dt}\cos\varphi \\ \omega_{y'} &= \frac{d\psi}{dt}\sin\vartheta\cos\varphi - \frac{d\vartheta}{dt}\sin\varphi \\ \omega_{z'} &= \frac{d\psi}{dt}\cos\vartheta + \frac{d\varphi}{dt} \end{aligned}\right\} \tag{11}$$

using these equations (LANDAU-LIFSIC, 1974), therefore we have the possibility to re-write the $\omega_{x'}$, $\omega_{y'}$, $\omega_{z'}$ angular velocity components in equations (4), (8) and (10) using the Euler-angles. Thus equations (4), (8) and (10) are:

$$\left.\begin{aligned} &A\left(\left(\frac{d\psi}{dt}\right)^2\sin^2\vartheta + \left(\frac{d\vartheta}{dt}\right)^2\right) + 2mgs\cos\vartheta = const. \\ &\frac{d\psi}{dt}\cos\vartheta + \frac{d\varphi}{dt} = \omega_{z'0} = const. \\ &A\frac{d\psi}{dt}\sin^2\vartheta + C\,\omega_{z'0}\cos\vartheta = const. \end{aligned}\right\} \tag{12}$$

where the *const.* value in the first equation already includes $C\omega_{z'0}^2$ as well. This is a set of three differential equations of first order, independent of each other, from these the three unknowns, $\vartheta(t)$, $\psi(t)$ and $\varphi(t)$ can be determined by numerical integration, i.e. the motion of the heavy symmetrical gyroscope can be given on the system $K$ fixed in space.

To solve the differential equation system (12) adequate initial conditions should be chosen at first. Let us chose as initial condition the case, when the heavy gyroscope's symmetry axis includes an angle of $\vartheta_0$ with the $z$ axis of vertical direction, and then let the gyroscope have a rotational angular velocity $\omega_{z'0}$ exclusively around its symmetry axis. I.e. at the moment $t = 0$:

$$\vartheta = \vartheta_0 \ , \quad \frac{d\vartheta}{dt} = 0 \ , \quad \frac{d\psi}{dt} = 0 \ , \quad \frac{d\varphi}{dt} = \omega_{z'0} \ .$$

Writing in (12) the values of constants corresponding to the initial conditions (in order: $2mgs\cos\vartheta_0$ , $\omega_{z'0}$ and $C\,\omega_{z'0}\cos\vartheta_0$ ), after transposition:

$$\left.\begin{aligned} &A\left[\left(\frac{d\psi}{dt}\right)^2 \sin^2\vartheta + \left(\frac{d\vartheta}{dt}\right)^2\right] = 2mgs(\cos\vartheta_0 - \cos\vartheta) \\ &\frac{d\psi}{dt}\cos\vartheta + \frac{d\varphi}{dt} = \omega_{z'0} \\ &A\frac{d\psi}{dt}\sin^2\vartheta = C\,\omega_{z'0}(\cos\vartheta_0 - \cos\vartheta) \end{aligned}\right\} \tag{13}$$

Expressing $d\psi / dt$ from the third equation of (13) and writing it in the first equation of (13), after a minor transposition:

$$\frac{d\vartheta}{dt} = \sqrt{\frac{\cos\vartheta_0 - \cos\vartheta}{A}\left(2mgs - \frac{C^2\omega_{z'0}^2}{A}\frac{(\cos\vartheta_0 - \cos\vartheta)}{\sin^2\vartheta}\right)}$$

A well approximating solution of this for the case of sufficiently quickly rotating gyroscopes (BUDÓ, 1964):

$$\vartheta = \vartheta_0 + \frac{A\,mgs\sin\vartheta_0}{C^2\omega_{z'0}^2}\left(1 - \cos\frac{C}{A}\omega_{z'0}\,t\right). \tag{14}$$

Introducing the following notations

$$\omega_{pr} = \frac{mgs}{C\omega_{z'0}} \quad \left(= \frac{M}{N_z\sin\vartheta}\right) \tag{15}$$

(average precessional angular velocity) and

$$\omega_{nu} = \frac{C}{A}\omega_{z'0} \tag{16}$$

(nutational angular velocity):

$$\boxed{\vartheta = \vartheta_0 + \frac{\omega_{pr}}{\omega_{nu}}\sin\vartheta_0\left(1 - \cos\omega_{nu}\,t\right)}. \tag{17}$$

Using this:

$$\frac{d\psi}{dt} = \omega_{pr}\left(1 - \cos\omega_{nu}\,t\right), \tag{18}$$

then integrating it for $t = 0$ with the $\psi = \psi_0$ initial condition:

$$\boxed{\psi = \psi_0 + \omega_{pr}\left(t - \frac{\sin \omega_{nu}\, t}{\omega_{nu}}\right)}. \tag{19}$$

Finally, from the second equation of (13) by substituting (18):

$$\boxed{\varphi = \omega_{z'0}\, t - \omega_{pr} \cos \vartheta_0 \left(t - \frac{\sin \omega_{nu}\, t}{\omega_{nu}}\right)}. \tag{20}$$

Summarizing the results obtained up to now, equations (17), (19) and (20) describe the motion of the heavy symmetric gyroscope in the inertial system $K$ fixed to the space that was given a spin at the moment $t = 0$ with an angular velocity of $\vec{\omega}$ and whose initial position corresponds to the Euler-angles of $\vartheta = \vartheta_0$, $\varphi = 0$, $\psi = 0$, $d\psi / dt = \omega_{z'0}$.

Equation (19) shows that the horizontal projection of the symmetry axis moves continuously around the $z$ axis with an average angular velocity of precession $\omega_{pr}$ determined by (18). According to (17) another motion also contributes to this, because the tilt angle $\vartheta$ included between the symmetry axis and the vertical direction fluctuates periodically between the initial $\vartheta_0$ and a slightly different value. The amplitude of this fluctuation (*nutation*) is $(\omega_{pr} / \omega_{nu}) \sin \vartheta_0$, and its angular velocity is $\omega_{nu}$. It can clearly be seen from the solution, that the precession is slower, on the other hand, the nutation is faster and its amplitude is lower, as the initial rotational angular velocity $\omega_{z'0}$ is higher. As according to (15) the angular velocity of precession is inversely proportional with the value of $\omega_{z'0}$, on the other hand, the amplitude of nutation according to (14) with the square of the value $\omega_{z'0}$, therefore in the case of sufficiently high rotational angular velocity, nutation can hardly be observed, precession is apparently regular (pseudo-regular precession). With suitable initial conditions rigorously regular precession is also possible. If the body rotates exactly round the symmetry axis, in the system $K'$ two components of the angular velocity vector are zero: $\omega_{x'} = \omega_{y'} = 0$, thus (4) can be written in a simpler form and the solution (17) simplifies to the form of $\vartheta = \vartheta_0$. In this case nutation does not accompany the precessional motion, this is the regular precession.

In the case of the free gyroscope no external torque exerts any effect, then in (15) because of $M = 0$ $\omega_{pr} = 0$, i.e. there is no precessional motion. In this case purely nutational motion can be experienced, when the body does not rotate round its symmetry axis.

## 4 Nutation of free gyroscopes

The main point of nutational motion can most simply be understood from the solution of Euler equations (7). In the case of a symmetrical free gyroscope, after substituting $A = B$ and $M_{x'} = M_{y'} = M_{z'} = 0$ in the $K'(x', y', z')$ system rotating together with the body:

$$\begin{bmatrix} \omega_{x'} \\ \omega_{y'} \\ \omega_{z'} \end{bmatrix} = \begin{bmatrix} m\cos\alpha \\ m\sin\alpha \\ \omega_{z'0} \end{bmatrix} \tag{21}$$

where

$$\alpha = \frac{C-A}{A}\omega_{z'0}\, t \tag{22}$$

is a linear function of time $t$, i.e. the vector $\vec{\omega}$ moves round the $z'$ axis of the coordinate system fixed to the mass of body (to the symmetry axis) with a constant angular velocity. Based on (22) the time of period belonging to a complete rotation of $\alpha = 2\pi$ depends exclusively on the rigid body's mass distribution, on the (C-A)/A dynamic oblateness.

During the nutational motion, as it can be seen in Fig. 3, the endpoint of the vector $\vec{\omega}$ describes a circle around the axis $z'$ with a constant angular velocity; its radius is $m = \sqrt{\omega_{x'}^2 + \omega_{y'}^2}$, thus the vector of rotational angular velocity itself moves along the surface of a circular cone whose vertex angle is $2\beta = 2arctg\,(m/\omega_{z'0})$ round the $z'$ coordinate axis that is identical with the major inertia axis $C$. Supposing the Earth to be a symmetrical rigid body, this is the *free nutation* referring to the Earth-fixed system $K'$.

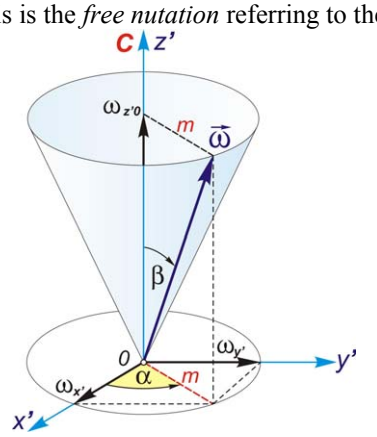


Fig. 3. Nutational motion viewed from the coordinate system fixed to the rotating body

## 5 The lunisolar precession

Let us apply our foregoing theoretical considerations to the case of Earth! At first, for the sake of simplicity, let us study only the impact of the torque arising from the gravitational attraction of Sun.

The Earth is in a dynamic equilibrium during its orbiting around the Sun; i.e. the $-\mathbf{F_K}$ centrifugal force of orbiting arising from Earth's orbiting around their common center of mass is in equilibrium with the $\mathbf{F_0}$ gravitational attraction of Sun acting on the mass center of Earth.

The Earth's shape is a rotational ellipsoid with good approximation; length of its equatorial half major axis is about 21 *km* longer than the length of half minor axis. Due

to the deviation from the spherical symmetrical mass distribution let us divide the Earth into an internal spherical symmetric mass domain and an Equatorial bulge as it can be seen in Fig. 4, then cut this bulge into two parts perpendicularly to the figure's plane. Be the center of mass of the bulge part closer to the Sun $P_1$, while that of the part being further $P_2$. In accordance with Newton's law of gravitational attraction the attraction force acting in $P_1$ is higher, but in $P_2$ it is lower than in the center of mass $O$. Since the centrifugal force of orbiting is the same in any point of the Earth due to the eccentric motion (VÖLGYESI, 1999), therefore the resultant of the two forces in the point $P_1$ is: $\mathbf{F} = \mathbf{F}_1 - \mathbf{F}_K$, and in point $P_2$ : $-\mathbf{F} = \mathbf{F}_2 - \mathbf{F}_K$. These two forces of equal magnitude but opposite direction result in the torque vector $\mathbf{M}$ perpendicular to the plane of Fig. 4.

Similarly to the Sun, the Moon also exerts torque on the Earth; moreover the torque generated by the Moon is significantly larger due to the proximity of Moon.

Result of the common effect of the torques generated this way is the precession motion of the Earth shown in Fig. 4, the so-called *lunisolar precession*.

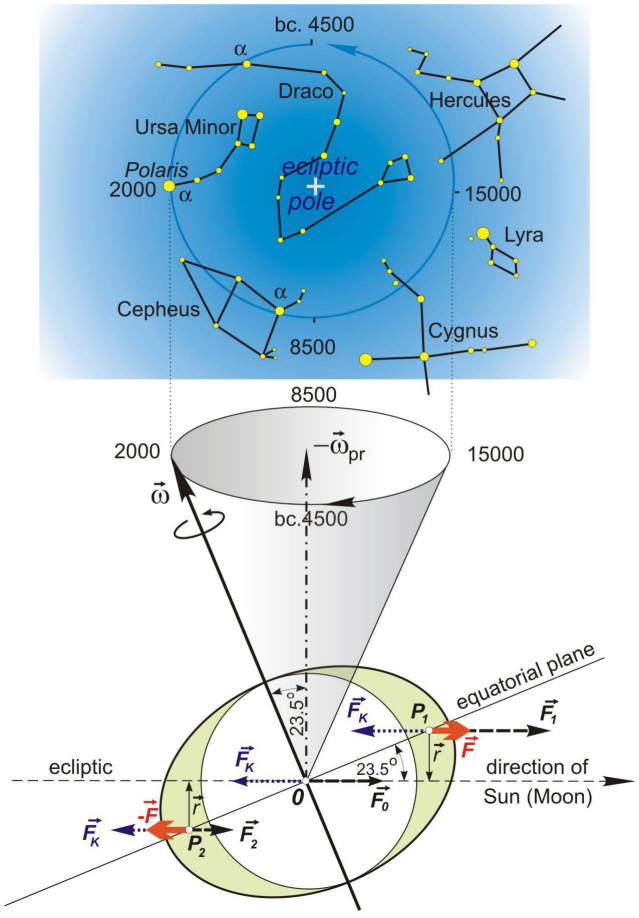


Fig. 4. Precesional motion of the Earth's rotation axis (lunisolar precession)

According to the astronomical observations the lunisolar precession manifests itself primarily in the fact that the Earth's rotation axis moves along the surface of a cone with a vertex angle of $2 \times 23.5 = 47^{o}$, in accordance with the angle of 23.5 included by the planes of ecliptic and the celestial equator; it makes a complete circuity in nearly 25 730 years. This according to Fig. 4 means, that the northern direction of the Earth's rotation axis pointed at the vicinity of the star $\alpha$ *Draconis* about 5000 years ago, at present the celestial pole is close to the $\alpha$ *Ursae Minoris* (Polaris) and after about 5000 years it will be close to the $\alpha$ *Cephei*. Thus, for the generations living now it is only a stroke of luck from the viewpoint of orientation by night that close to place of the celestial north pole a relatively bright star, the North Star can be found.

## 6 The planetary precession

Under the effect of the solar system's planets the Earth's orbital plane slowly changes relative to the average orbital plane of planets, therefore the tilt angle included by the Equator's and the ecliptic's plane fluctuates between about $22^{o}$ and $24.5^{o}$ with an almost 40000 year period. Thus, the normal of ecliptic wanders round along the surface of a circular cone whose vertex angle is about $2.5^{o}$, with a time of period nearly 40000 years. Since the ecliptic's normal is the axis of precession cone, therefore the cone of lunisolar precession sometimes slightly opens, sometimes slightly closes due to the oscillation of the ecliptic's plane with a period of 40000 years; i.e. the vertex angle of the cone is not steadily $47^{o}$, but it varies with a period of approximately 40000 years between about 44 and $49^{o}$.

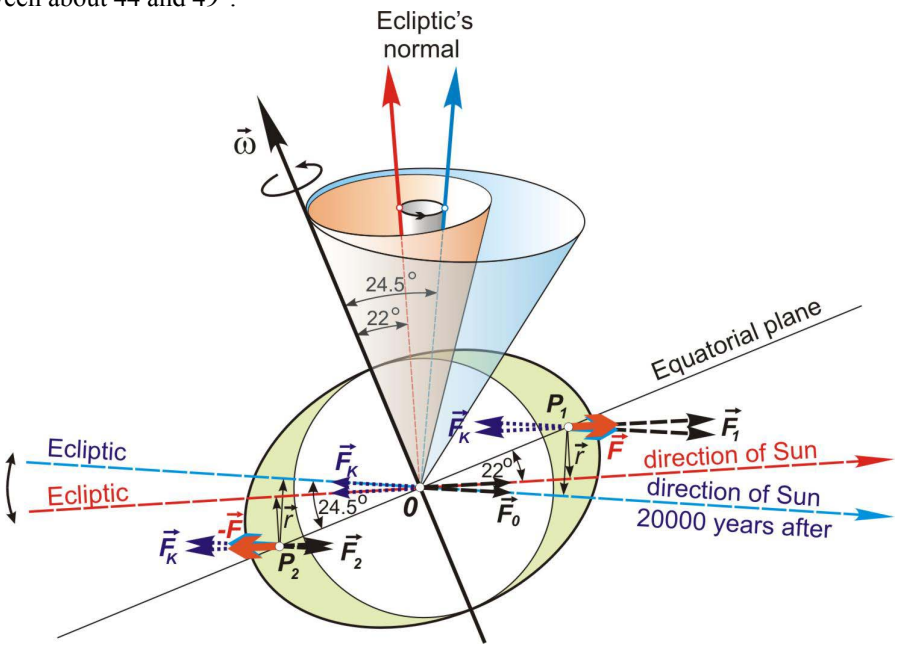


Fig. 5. The planetary precession

As a matter of fact, due to the motion of the ecliptic's plane the Sun and the Moon can be seen from the Earth in continuously other directions, with a period of 40000 years; and therefore the separation between the part-centers of mass $P_1$ and $P_2$, and the ecliptic's plane changes continuously as shown in Fig. 5, i.e. the torque generation the precession motion changes (fluctuates) continuously because the lever arm continuously changes.

The resultant of the lunisolar and planetary precession motions is the general precession, by other name the *normal precession*. During the normal precessional motion the celestial pole moves not exactly along a circular path due to the oscillation of the ecliptic's pole, but it wanders related to the fixed stars along a curve, which closely approximates the circular path although *the curve does not return into itself*. Under the effect of the normal precession the vernal point shifts westward along the ecliptic with a value of about $50.26''$ annually, consequently the time of a complete circuity is about 25786 years.

## 7 Disturbing precession

As a consequence of the Moon's, Sun's and planets' motion related to the Earth a torque changing in time has effect on Earth, therefore different fluctuations of shorter period are added to the normal precession motion. These short period changes in the precession motion of the rotation axis have been incorrectly called *astronomical nutation* up to now; in what follows this phenomenon is called *disturbing precession*. The disturbing precession comprises motions of different period and amplitude and it is superposed on the long period (secular) precession motion.

It has two rather important periods due to the changes in the relative position of the Sun and the Earth. The magnitude of the torque exerted by the Sun on the equatorial bulge of Earth depends on the Sun's angle of declination (on the altitude above the Earth's equatorial plane). Fig. 4 e.g. shows the Earth in the position of winter solstice, when $\delta = -23.5^o$. At that time and on the day of summer solstice (when $\delta = +23.5^o$) the Sun exerts maximum torque on the Earth. Between the two positions the torque decreases and increases, respectively. At the moment of the spring and fall equinox the centers of gravity $P_1$ and $P_2$ of the Earth's two equatorial excess masses (equatorial bulge) as interpreted in Fig. 4 are at equal distances from the Sun, thus the torque causing the precession is zero. In accordance with this, the Earth's precession varies with a half year period, due to the changes in the Sun's declination. A precession variation of one-year period is also added to this; this is a consequence of the fact that the Earth orbits in an orbit of elliptical shape around the Sun and therefore its distance measured from the Sun varies with a one-year period, and in accordance with this the torque as well.

During its orbit around the Earth the Moon causes completely similar changes, but its periods are shorter and amplitudes are higher. In addition, motion of the Moon has a further effect too, that is even much more important than the previous ones. The Moon does not orbit around the Earth in one and the same plane in which the Earth orbits around the Sun, the Moon's orbital plane and the plane of ecliptic include an angle of nearly $5^o09'$. As the intersection line of the Moon's orbital plane and the ecliptic plane (the nodal line of the Moon's orbit) wanders round in the plane of ecliptic with a period

of 18.6 years, therefore the Moon can be seen from the Earth in continuously different directions with a period of 18.6 years, and thus according to Fig. 6 continuously changes the distance of the part-centers of mass $P_1$ and $P_2$ from the Moon's orbital plane. As a consequence, the torque causing the precession motion also changes (fluctuates) continuously, because the lever arm continuously changes.

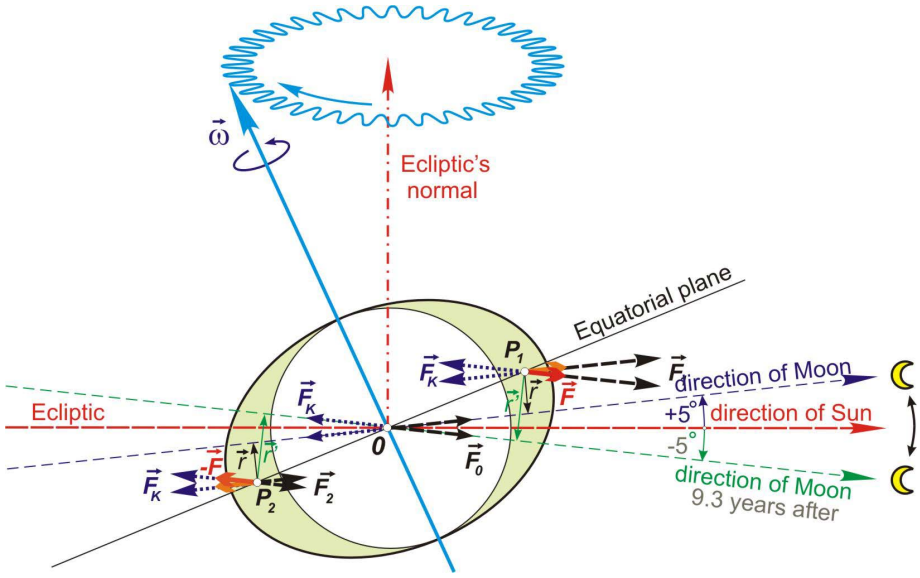


Fig. 6. Impact of the lunar main component of disturbing precession

The disturbing precession's component which originates from the motion of the Moon orbit's nodal line is many times larger than together all the other changes comprising the precession, therefore it is called the *lunar main component of the disturbing precession*.

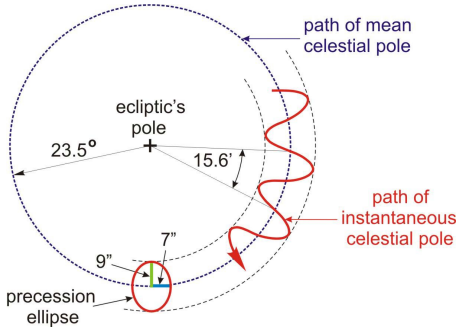


Fig. 7. The precession ellipse

Thus the Earth's rotational angular velocity vector wanders round the normal of the ecliptic with an average vertex angle of about $47^{o}$ at present, along the wavy surface of a cone that can be seen in Fig. 6. These waves have amplitudes of about $9''$ (so much is

the fluctuation in the tilt of the rotation axis: the so-called obliquity component), and their wavelengths are nearly $15.6'$.

It is a general practice as well to demonstrate the precession motion together with the main component of the disturbing precession with the so-called precession ellipse that can be seen in Fig. 7. (The term of nutation ellipse widely spread in practice up to now is incorrect because this has nothing to do with nutation.) Thus, the center of the precession ellipse wanders with constant velocity round the ecliptic's pole at a polar distance of $23.5^{o}$ and makes a complete circle in 26000 years, while the real (the instantaneous) celestial pole move along the precession ellipse with a period of 18.6 years. The half major axis of the precession ellipse whose distance is $9''$ is always directed toward the ecliptic's pole, and its half minor axis of a $7''$ distance is perpendicular to that.

## Summary

The spatial direction and magnitude of the angular velocity vector always changes during the Earth's rotation. The changes can be seen summarized in Fig. 8. Change in the absolute value (length of the day) of the angular velocity vector $\vec{\omega}$ is primarily the consequence of the so-called tidal friction caused by the Moon and the Sun (VÖLGYESI, 1999). Changes in spatial direction of the angular velocity vector $\vec{\omega}$ can be divided into two groups: changes caused by the precession and nutation motions. The two components of the precession motion are the normal precession and the disturbing precession, and two further components of the normal precession are the lunisolar and the planetary precession. The Earth's nutational motion, also consists of two components: the polar motion and the polar wandering; while the polar motion has two further components, the free nutation and the forced nutation.

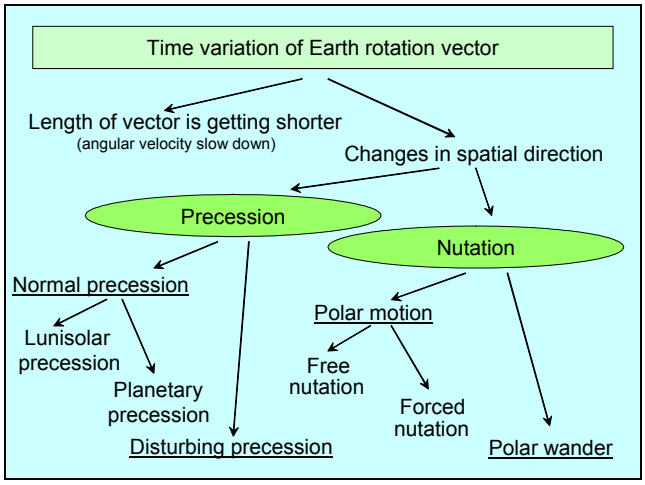


Fig. 8. Variation of the Earth's rotational angular velocity vector in time and space

From the above considerations it can clearly be seen that because of the relative changes in the positions of the Moon, the Sun and the planets related to the Earth the fluctuations of shorter period arising as a result of the time-varying torque cannot be

called nutation in physical sense. Due to the complicated movement of the relevant celestial bodies the torque generating the precession changes, as a consequence the mass of the Earth together with the rotation axis of fixed position relative to it makes the *varying* precession motion. Contrary to this, during the nutational motion the rotation axis of the Earth does not move together with the Earth's mass, but independently of the impact of any kind of torque the Earth's mass and its symmetry axis separately from the rotation axis make their own complicated motions (this is merely due to the fact that the rotation does not take place round the symmetry axis). Wording it in a simpler form *extraterrestrial masses are responsible for the precession, on the other hand, exclusively the Earth's own mass (its mass distribution) is responsible for the nutation.*

Consequently, it seems to be reasonable to use the term of *disturbing precession* in the future instead of the misleading *astronomical nutation* (VÖLGYESI 2003a, 2003b, 2003c).

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